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Minimal Lepton Flavor Violation and Renormalization Group Evolution of Lepton Masses and Mixing

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Abstract

We study the renormalization group equations (RGEs) of the neutrino parameters in models of Minimal Lepton Flavor Violation. In such models, the RGEs can be described in terms of flavor spurions, such that only the coefficients depend on the specific model. We explicitly demonstrate this method for the SM and MSSM for both Type-I and Type-III seesaw models. For that purpose, the RGEs of neutrino parameters in the MSSM Type-III seesaw have been computed. We have extended this method to get the evolution equations at second order. The implications for leptogenesis are also discussed.

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I. INTRODUCTION

The data from past and ongoing neutrino oscillation experiments, as well as from cosmology and astrophysics, have now confirmed that neutrinos have distinct masses and that the three neutrino flavors ν_e, ν_μ and ν_τ mix among themselves to form the three mass eigenstates. The fact that the neutrinos are massive and mix implies non-conservation of lepton flavor. Hence, lepton flavor violating processes are expected in the lepton sector just as quark flavor violating processes arise in the quark sector.

In the quark sector of the Standard Model (SM), flavor violation is induced by the Yukawa matrices such that baryon number remains an exact symmetry. The fact that flavor changing neutral currents (FCNCs) are heavily suppressed puts stringent constraints on the possible structure of new degrees of freedom carrying flavor quantum numbers. These constraints can be satisfied either if new particles are very heavy or if flavor symmetries suppress the flavor changing couplings. One of the most predictive and restrictive symmetry principles that can be used is Minimal Flavor Violation (MFV) [1]. The MFV framework is the assumption that in the quark sector the only sources of flavor symmetry breaking are the Yukawa couplings.

While the idea of MFV has a straightforward and unique realization in the quark sector, the situation is different in the lepton sector. The reason is that the neutrinos can be Majorana particles, in which case total lepton number is no longer a symmetry of the theory. Due to this complication, the Minimal Lepton Flavor Violation (MLFV) hypothesis is not uniquely defined; there are two ways to define it [2]. In the first case, known as MLFV with minimal field content, we do not add any new field to the theory, and treat the neutrino mass terms as non-renormalizable terms. The only irreducible sources of lepton flavor violation are the charged-lepton Yukawa matrix and the effective left-handed Majorana mass matrix. The breaking of total Lepton Number (LN) is independent of the flavor violation and happens at some very high scale.

The other possibility, called MLFV with extended field contents (MLFV-ex), is to introduce new fields to the SM. In particular, three heavy right-handed neutrinos are added to the SM. Their Majorana mass term, which is assumed to be flavor universal, explicitly breaks LN. In this scenario, the two Yukawa matrices act as the only irreducible sources of flavor violation. In the MLFV-ex scenario, the low energy observables depend on the high energy parameters of the theory. For example, the FCNC constraints in the leptonic sector affects leptogenesis. This has been studied in [3] with the mass-splitting of the right-handed neutrinos, required for successful leptogenesis, being introduced from flavor symmetry considerations only. To have a complete understanding of the relation between the high energy parameters and the low energy observables, one needs to study the complete renormalization group (RG) evolution effects in this context. RG evolution has already been shown to have strong effects on leptogenesis [4]. Ref. [5] shows the stability of the MLFV under

RG evolution in the context of soft masses in the Minimal Supersymmetric Standard Model (MSSM). While [6] takes into account the RG evolution effects in the context of $\mu \to 3e$ and $\tau \to 3\ell$ decays, a general analysis of RG evolution of lepton masses and mixing parameters in the MLFV framework is still lacking.

In this paper, we consider the RG evolution of lepton masses and mixing parameters in the MLFV-ex scenario, with the SM as the low energy effective theory. The basic idea is that the RGEs can be written in terms of spurions that depend only on the Yukawa matrices. The coefficient of each term can be model dependent. Moreover, we assume that the universality of the Majorana masses is broken slightly, and hence treat the Majorana mass matrix as a spurion of our theory, and we get the RGE for this spurion as well. This is, in fact, a natural assumption as the universality is automatically broken in course of RG evolution. We show explicitly how one can write the RGEs for the SM and MSSM in both Type-I and Type-III seesaw models. The advantage of the spurion formalism is that it shows how each combination enters and can be used as a check for any MLFV model.

II. THE MODEL: νMSM AND MLFV WITH EXTENDED FIELD CONTENT

We consider the SM extended by three right-handed neutrinos, which are singlets under the SM gauge group. This model is referred to as the νMSM [7]. We also consider the case where they are triplet under the $\mathrm{SU}(2)_L$ group later in this section.

We begin by considering the model excluding all mass terms of the leptons and gradually introduce mass terms to study their effect on the flavor symmetries of the theory, at different energy scales μ. In the massless lepton limit, at high scale $\mu > M_R$, the νMSM enjoys a flavor symmetry $\mathrm{G}^0_{\mathrm{LF}}$, similar to that of the quark sector, given by

$$\mathrm{G}^0_{\mathrm{LF}} = \mathrm{SU}(3)_{l_L} \otimes \mathrm{SU}(3)_{e_R} \otimes \mathrm{SU}(3)_{\nu_R} \ . \tag{2.1}$$

Here we consider only the non-Abelian part of the flavor symmetry group. This sector is also invariant under U(1) of hypercharge (Y), total lepton number (LN), as well as $\mathrm{U}(1)_{\mathrm{E}}$ (or $\mathrm{U}(1)_\nu$), which corresponds to a rotation of the e_R (or ν_R) fields.

The presence of Majorana mass term for the right-handed neutrinos reduces the symmetry. Let us denote the right-handed neutrinos by ν_R^i, $i \in \{1, 2, 3\}$. The only source of LN violation in this model is the Majorana mass term of these right-handed neutrinos given by

$$\mathcal{L}_{\mathrm{Maj}} = -\frac{1}{2}\bar{\nu}_R^C M_\nu \nu_R + \text{h.c.} \ , \tag{2.2}$$

where C denotes charge conjugation. The right-handed Majorana mass matrix M_ν is symmetric, $M_\nu = M_\nu^T$. Furthermore, without any loss of generality, we can choose M_ν to be real by re-definition of the phases of ν_R^i. (The νMSM was originally defined [7] in the basis where the charged lepton mass matrix and the Majorana mass matrix are real and diagonal.) In

general M_ν breaks $\mathrm{SU}(3)_{\nu_R}$ completely. For a universal mass matrix, however, the breaking is into an O(3) group. In this case, the Majorana mass matrix is given by

$$\left(M_\nu\right)_{ij} = M_R \delta_{ij} \;, \tag{2.3}$$

and the flavor symmetry group becomes

$$\mathrm{G}_{\mathrm{LF}}^0 \to \mathrm{G}_{\mathrm{LF}} = \mathrm{SU}(3)_{l_L} \otimes \mathrm{SU}(3)_{e_R} \otimes \mathrm{O}(3)_{\nu_R} \;. \tag{2.4}$$

The two Yukawas Y_e and Y_ν are given by

$$\mathcal{L}_{\mathrm{Yukawa}} = -\bar{e}_R Y_e \phi^\dagger l_L - \bar{\nu}_R Y_\nu \widetilde{\phi}^\dagger l_L + \mathrm{h.c.} \;, \tag{2.5}$$

where ϕ is the SM Higgs doublet and $\widetilde{\phi} = i\sigma^2\phi^*$, σ^2 being the second Pauli matrix.

It is customary to treat G_{LF} as an unbroken symmetry of the underlying theory which can be achieved by treating the Yukawa matrices as spurion fields with non-trivial quantum numbers under G_{LF}

$$Y_e \sim (\bar{3}, 3, 1) \;, \qquad Y_\nu \sim (\bar{3}, 1, 3) \;. \tag{2.6}$$

The νMSM in the massless lepton limit and with universal right-handed Majorana masses enjoys the flavor symmetry G_{LF} and this is the MLFV hypothesis with extended field content (MLFV-ex) [2]. Going beyond the MLFV-ex hypothesis, in this paper we choose the universality of M_ν to be slightly broken, which happens also as a result of RG evolution. We thus treat M_ν as a spurion transforming, under G_{LF}, as

$$M_\nu \sim (1, 1, 6) \;. \tag{2.7}$$

The spurions have the following transformation properties:

$$Y_e \to U_R Y_e U_L^\dagger \;, \qquad Y_\nu \to O_\nu Y_\nu U_L^\dagger \;, \qquad M_\nu \to O_\nu M_\nu O_\nu^T \;, \tag{2.8}$$

where $U_L \in \mathrm{SU}(3)_{l_L}$, $U_R \in \mathrm{SU}(3)_{e_R}$ and $O_\nu \in \mathrm{O}(3)_{\nu_R}$. This technique is known as spurion analysis.

Finally, the heavy fields generate small neutrino masses via the seesaw relation [8]

$$m_\nu = \frac{v^2}{2} Y_\nu^T M_\nu^{-1} Y_\nu \;, \tag{2.9}$$

where the vacuum expectation value of the SM Higgs is defined as $\langle\phi\rangle = (0, v/\sqrt{2})^T$. In the MLFV-ex model, the left-handed neutrino mass matrix is given by

$$m_\nu|_{\mathrm{MLFV-ex}} = \frac{v^2}{2M_R} Y_\nu^T Y_\nu \;. \tag{2.10}$$

Note that in general $Y_\nu^T Y_\nu$ and $Y_\nu^\dagger Y_\nu$ are two different sources of G_{LF} breaking. Only in the limit where Y_ν is real are they the same [2]. We do, however, expect to have CP violation in

the theory and thus we do not concentrate on the case of real Y_ν. We consider the MLFV-ex model for $\mu > M_R$ energy regime for the rest of the paper, with the exception that the universality of M_ν is assumed to be slightly broken. We consider the case where M_R is large compared to the electroweak symmetry breaking scale. This ensures that $\mathrm{U}(1)_{\mathrm{LN}}$ is broken at some high scale, and that, in general, the breaking of LN by the Majorana mass term is independent of G_{LF}-violation.

Next, we discuss the effective theory below M_R, or equivalently below the scale of the lightest of the heavy right-handed neutrinos, when universality is broken. In this regime, all the three heavy right-handed neutrinos get integrated out, and as a result the flavor symmetry group reduces to

$$\mathrm{G}_{\mathrm{LF}} \to \mathrm{G}'_{\mathrm{LF}} = \mathrm{SU}(3)_{l_L} \otimes \mathrm{SU}(3)_{e_R} \ . \tag{2.11}$$

In this energy region, the dimension-5 non-renormalizable term in the Lagrangian responsible for the LN-violating left-handed Majorana neutrino masses is of the form

$$\mathcal{L} \sim \bar{l}_L^C m_\nu l_L \phi\phi \ . \tag{2.12}$$

There are two sources of $\mathrm{G}'_{\mathrm{LF}}$ breaking in this case. The charged lepton Yukawa Y_e and the left-handed neutrino mass m_ν that transform as

$$Y_e \sim (\bar{3}, 3) \ , \quad m_\nu \sim (6, 1) \ . \tag{2.13}$$

Thus, the model becomes equivalent to the MLFV hypothesis with minimal field content [2]. In this case, m_ν remains the only relevant quantity that contains the high energy information of the neutrino parameters, which in turn can be extracted by the measurement of the neutrino masses and mixing parameters. Hence, the effect of RG evolution becomes an important factor to be taken into account, which we will be studying in the following sections.

In the framework of spurion analysis, G_{LF} is broken by the background values of the spurions. We consider the background values of $Y_{e,\nu}$ to be small, the largest one being experimentally measured to be $Y_\tau \sim 0.01$ at the scale M_Z. Thus, we can use perturbation theory and consider only the leading order corrections. To first order, the operators responsible for the breaking of G_{LF} are combinations of two Yukawa matrices, that is, working at one loop is equivalent of considering spurions with two couplings. There are several combinations of couplings that can appear in the result. These couplings and their transformation properties are given in Table I. As can be seen, M_ν appears only when we consider the evolution of M_ν itself.

The flavor symmetry structure is more complicated when the heavy neutrinos are not exactly degenerate. A breaking of the universality of M_ν, however small, is also necessary for leptogenesis as has been shown in [3]. In that paper, the degeneracy is broken by appropriate combinations of spurions in the MLFV-ex scenario. Our assumption is that the

Combination of spurions	Transformation
$Y_e^\dagger Y_e$	$(8 \oplus 1, 1, 1)$
$Y_e Y_e^\dagger$	$(1, 8 \oplus 1, 1)$
$Y_\nu^\dagger Y_\nu$	$(8 \oplus 1, 1, 1)$
$Y_\nu Y_\nu^\dagger$	$(1, 1, 8 \oplus 1)$
$\mathrm{Tr}[Y_e^\dagger Y_e] = \mathrm{Tr}[Y_e Y_e^\dagger]$	$(1, 1, 1)$
$\mathrm{Tr}[Y_\nu^\dagger Y_\nu] = \mathrm{Tr}[Y_\nu Y_\nu^\dagger]$	$(1, 1, 1)$
$T_e \equiv Y_e^\dagger Y_e - \frac{1}{3}\mathrm{Tr}[Y_e^\dagger Y_e]\mathbf{I}_3$	$(8, 1, 1)$
$T'_e \equiv Y_e Y_e^\dagger - \frac{1}{3}\mathrm{Tr}[Y_e Y_e^\dagger]\mathbf{I}_3$	$(1, 8, 1)$
$T_\nu \equiv Y_\nu^\dagger Y_\nu - \frac{1}{3}\mathrm{Tr}[Y_\nu^\dagger Y_\nu]\mathbf{I}_3$	$(8, 1, 1)$
$T'_\nu \equiv Y_\nu Y_\nu^\dagger - \frac{1}{3}\mathrm{Tr}[Y_\nu Y_\nu^\dagger]\mathbf{I}_3$	$(1, 1, 8)$

TABLE I: Transformations of combinations of two spurion fields under $\mathrm{G_{LF}}$. We have used the SU(3) algebra $3 \otimes \bar{3} = 8 \oplus 1$.

amount of non-degeneracy is small and $\mathrm{G_{LF}}$ is still the flavor symmetry of the underlying theory. The effect of the breaking is due to the fact that running at the scale in between the three masses is not described by any of the two regions we discussed above. Yet, if the breaking is small this running is not significant and integrating out all the neutrinos together is a good approximation. Moreover, if the degeneracy is lifted due to RG evolution, then taking it into account is formally a higher order effect.

III. RG EVOLUTION OF NEUTRINO PARAMETERS

We now study the effect of RG running. At energy scales above M_R, the quantities of interest are the Yukawa matrices $Y_{e,\nu}$, and the right-handed neutrino mass matrix M_ν. Below, we see how they run.

In all our discussions, we consider only one loop running. In term of spurions, each loop add two Yukawa terms, and thus working at one loop is done by using only terms that have two Yukawa couplings more than the tree level one. The evolution equations at second order are discussed in Appendix B.

A. RG evolution of Y_e

We define

$$\dot{Y}_e \equiv \frac{dY_e}{dt}\,, \qquad t \equiv \frac{\ln(\mu/\mu_0)}{16\pi^2}\,. \tag{3.1}$$

Here $\mu_0(> M_R)$ is some high energy scale at which we start running and the factor $(16\pi^2)$ appears because of the fact that we consider radiative corrections at 1-loop.

Under the flavor symmetry group $\mathrm{G_{LF}}$, Y_e transforms as $(\bar{3}, 3, 1)$ and so does $\dot{Y}_e$. Hence $\dot{Y}_e$ can be expressed as appropriate combinations of the spurion fields transforming as $(\bar{3}, 3, 1)$. Table I shows the combinations of two spurion fields with their transformation properties. Using the SU(3) algebra

$$8 \otimes \bar{3} = \overline{15} \oplus 6 \oplus \bar{3} \,, \qquad 8 \otimes 3 = 15 \oplus \bar{6} \oplus 3 \,, \tag{3.2}$$

and we can write

$$\begin{aligned} Y_e T_e &= (\bar{3}, 3, 1) \otimes (8, 1, 1) \ni (\bar{3}, 3, 1) \,, & (3.3) \\ Y_e T_\nu &= (\bar{3}, 3, 1) \otimes (8, 1, 1) \ni (\bar{3}, 3, 1) \,, & (3.4) \\ Y_e \mathrm{Tr}[Y_e^\dagger Y_e] &= (\bar{3}, 3, 1) \otimes (1, 1, 1) = (\bar{3}, 3, 1) \,, & (3.5) \\ Y_e \mathrm{Tr}[Y_\nu^\dagger Y_\nu] &= (\bar{3}, 3, 1) \otimes (1, 1, 1) = (\bar{3}, 3, 1) \,. & (3.6) \end{aligned}$$

The above combinations are the only terms, containing three spurion fields, allowed to appear on the right-hand side (RHS) of the RGE for $\dot{Y}_e$. $T_e' Y_e$ gives the same term as that given by $Y_e T_e$ and so has not been listed separately. Thus, at 1-loop, when terms up to combinations of three spurion fields are allowed, the most general form of $\dot{Y}_e$ is given by

$$\begin{aligned} \dot{Y}_e &= \widetilde{a}_1 Y_e T_e + \widetilde{a}_2 Y_e T_\nu + \widetilde{a}_3 Y_e \mathrm{Tr}[Y_e^\dagger Y_e] + \widetilde{a}_4 Y_e \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + a_5 Y_e \\ &= Y_e \left(a_1 Y_e^\dagger Y_e + a_2 Y_\nu^\dagger Y_\nu + a_3 \mathrm{Tr}[Y_e^\dagger Y_e] + a_4 \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + a_5 \mathbb{1}_3 \right) \,, \end{aligned} \tag{3.7}$$

where a_1, a_2, a_3, a_4 and a_5 are expected to be numbers of $\mathcal{O}(1)$ that can be determined by the calculation of the 1-loop diagrams in the theory.

The case of a_5 is a bit more involved since it is a function independent of spurion fields. Thus a_5 must contain combinations of other couplings in the theory that transform trivially under $\mathrm{G_{LF}}$. The couplings that we have in the theory are the gauge couplings, g_i, the Higgs self-coupling, λ, and the quark Yukawa couplings $Y_{U,D}$. Since leptons are singlets under $\mathrm{SU}(3)_C$, g_3 cannot contribute. Moreover, at 1-loop the Higgs self-coupling cannot contribute either. Terms proportional to g_1 and g_2 contributing to a_5 must be of form

$$a_{g_1} g_1^2 + a_{g_2} g_2^2 \,. \tag{3.8}$$

The singlet combination made of the quark Yukawas $Y_{U,D}$ is of the form $\mathrm{Tr}[Y_i^\dagger Y_i]$, and the most general form of the quark Yukawa contributions to a_5 is

$$a_U \mathrm{Tr}[Y_U^\dagger Y_U] + a_D \mathrm{Tr}[Y_D^\dagger Y_D]. \tag{3.9}$$

Thus the general form of a_5 is given by

$$a_5 = a_{g_1} g_1^2 + a_{g_2} g_2^2 + a_U \mathrm{Tr}[Y_U^\dagger Y_U] + a_D \mathrm{Tr}[Y_D^\dagger Y_D] \,, \tag{3.10}$$

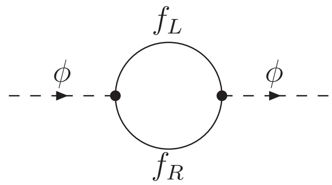

FIG. 1: The self-energy diagram of the Higgs ϕ with complete fermion loop, where the fermion pair $\{f_L, f_R\}$ can be $\{l_L, e_R\}$, $\{l_L, \nu_R\}$, $\{q_L, u_R\}$ or $\{q_L, d_R\}$ producing contributions proportional to $\mathrm{Tr}[Y_e^\dagger Y_e]$, $\mathrm{Tr}[Y_\nu^\dagger Y_\nu]$, $\mathrm{Tr}[Y_U^\dagger Y_U]$ and $\mathrm{Tr}[Y_D^\dagger Y_D]$ respectively.

and the general form of $\dot{Y}_e$ becomes

$$\begin{aligned}\dot{Y}_e &= Y_e\left(a_1 Y_e^\dagger Y_e + a_2 Y_\nu^\dagger Y_\nu\right) + Y_e\left(a_{g_1} g_1^2 + a_{g_2} g_2^2\right) \\ &+ Y_e\Big(a_3 \mathrm{Tr}[Y_e^\dagger Y_e] + a_4 \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + a_U \mathrm{Tr}[Y_U^\dagger Y_U] + a_D \mathrm{Tr}[Y_D^\dagger Y_D]\Big) \ . \end{aligned} \tag{3.11}$$

Terms proportional to $\mathrm{Tr}[Y_x^\dagger Y_x]$ ($x \in \{e, \nu, U, D\}$) arise from a complete fermion loop in the self-energy correction of the scalar Higgs boson, as shown in Fig. 1. Since quarks come in three colors, one gets

$$a_3 : a_4 : a_U : a_D = 1 : r : 3 : 3, \tag{3.12}$$

where each of the three quark colors contributes equally. $r \equiv a_4/a_3$ is determined by the transformation properties of the right-handed neutrinos under the gauge group. As we discuss below, at Eq. (4.3), for singlets $r = 1$, while for triplets $r = 3$. We can now define a quantity

$$T \equiv \mathrm{Tr}[Y_e^\dagger Y_e] + r\mathrm{Tr}[Y_\nu^\dagger Y_\nu] + 3\mathrm{Tr}[Y_U^\dagger Y_U] + 3\mathrm{Tr}[Y_D^\dagger Y_D] \ , \tag{3.13}$$

and write $\dot{Y}_e$ in a simpler form as

$$\dot{Y}_e = Y_e\left(a_1 Y_e^\dagger Y_e + a_2 Y_\nu^\dagger Y_\nu\right) + Y_e\left(a_T T + a_{g_1} g_1^2 + a_{g_2} g_2^2\right) \ , \tag{3.14}$$

where a_T, a_{g_1}, a_{g_2}, are expected to be of $\mathcal{O}(1)$.

B. RG evolution of Y_ν

Next, we discuss the running of Y_ν. Since Y_ν transforms as $(\bar{3}, 1, 3)$ under $\mathrm{G_{LF}}$, so must be $\dot{Y}_\nu$. From Table I and using Eq. (3.2) we obtain that the only allowed combinations of spurion fields at one loop order are

$$Y_\nu T_e = (\bar{3},1,3) \otimes (8,1,1) \ni (\bar{3},1,3) \ , \tag{3.15}$$

$$Y_\nu T_\nu = (\bar{3},1,3) \otimes (8,1,1) \ni (\bar{3},1,3) \ , \tag{3.16}$$

$$Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e] = (\bar{3},1,3) \otimes (1,1,1) = (\bar{3},1,3) \ , \tag{3.17}$$

$$Y_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu] = (\bar{3},1,3) \otimes (1,1,1) = (\bar{3},1,3). \tag{3.18}$$

$T'_\nu Y_\nu$ gives the same term as that given by $Y_\nu T_\nu$ and so has not been written here. Finally, as with $\dot{Y}_e$, we can write

$$\dot{Y}_\nu \;=\; \widetilde{b}_1 Y_\nu T_e + \widetilde{b}_2 Y_\nu T_\nu + \widetilde{b}_3 Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e] + \widetilde{b}_4 Y_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + \widetilde{b}_5 Y_\nu \;, \tag{3.19}$$

which can be simplified, using a similar approach to that of the previous section, to get

$$\dot{Y}_\nu \;=\; Y_\nu \left(b_1 Y_e^\dagger Y_e + b_2 Y_\nu^\dagger Y_\nu\right) + Y_\nu \left(b_T T + b_{g_1} g_1^2 + b_{g_2} g_2^2\right) \;, \tag{3.20}$$

where T is defined in Eq. (3.13), and b_1, b_2, b_T, b_{g_1}, b_{g_2}, are expected to be of $\mathcal{O}(1)$.

C. RG evolution of the heavy right-handed Majorana mass M_ν

Once we know the evolution of the Yukawa matrices, we can discuss the running of the physical masses. We consider the right handed neutrino mass term

$$\mathcal{L}_{\mathrm{Maj}} = -\frac{1}{2}\left[\bar{\nu}_R^C M_\nu \nu_R + \bar{\nu}_R M_\nu^\dagger \nu_R^C\right] \;. \tag{3.21}$$

We first discuss the evolution of M_ν below and later consider $M_\nu^\dagger$.

As already stated, M_ν transforms as (1,1,6) under $\mathrm{G_{LF}}$ and thus is symmetric under $\mathrm{O(3)}_{\nu_R}$. Hence while considering the RG evolution of M_ν, the RHS must contain terms which has the same transformation properties under $\mathrm{G_{LF}}$. Using the transformation rules in Table I and the SU(3) algebra

$$6 \otimes 8 = 24 \oplus \overline{15} \oplus 6 \oplus \bar{3} \;, \tag{3.22}$$

the allowed terms are obtained to be

$$\begin{aligned}
M_\nu T'_\nu \;&=\; (1,1,6) \otimes (1,1,8) \ni (1,1,6) \;, && (3.23)\\
M_\nu \mathrm{Tr}[Y_e^\dagger Y_e] \;&=\; (1,1,6) \otimes (1,1,1) = (1,1,6) \;, && (3.24)\\
M_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu] \;&=\; (1,1,6) \otimes (1,1,1) = (1,1,6) \;. && (3.25)
\end{aligned}$$

The quark Yukawas, $Y_{U,D}$, are expected to have contributions of form $\mathrm{Tr}[Y_i^\dagger Y_i]$ ($i \in \{U, D\}$). In general, there will also be terms containing g_i^2 and λ.

To get the final form of the Y_ν dependence of $\dot{M}_\nu$ we have to take into account the fact that M_ν is symmetric. Symmetrizing we obtain

$$\begin{aligned}
&\frac{1}{2}\left[\left(M_\nu Y_\nu Y_\nu^\dagger\right)^{\alpha\beta} + \left(M_\nu Y_\nu Y_\nu^\dagger\right)^{\beta\alpha}\right] + \frac{1}{2}\left[(M_\nu)^{\alpha\beta}\mathrm{Tr}[Y_\nu^\dagger Y_\nu] + (M_\nu)^{\beta\alpha}\mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right]\\
=\;& \frac{1}{2}\left[\left(M_\nu Y_\nu Y_\nu^\dagger\right)^{\alpha\beta} + \left(\left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\right)^{\alpha\beta}\right] + (M_\nu)^{\alpha\beta}\mathrm{Tr}[Y_\nu^\dagger Y_\nu] \;,
\end{aligned}$$

where α, β are $\mathrm{O}(3)_{\nu_R}$ indices. We can then write the most general form of the RG equation for M_ν as

$$\dot{M}_\nu = \frac{q_1}{2}\left[M_\nu(Y_\nu Y_\nu^\dagger) + \left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\right] + M_\nu\left(q_T T + q_{g1} g_1^2 + q_{g2} g_2^2 + q_{g3} g_3^2 + q_\lambda \lambda\right) , \quad (3.26)$$

where T is given by Eq. (3.13). All of q_is are expected to be of $\mathcal{O}(1)$. As already discussed, the trace term T can appear only through Higgs interactions, and so it cannot be present here since M_ν does not couple to Higgs rendering $q_T = 0$. Moreover, since the added lepton fields ν_R^i are singlets under $\mathrm{U}(1)_Y$ and $\mathrm{SU}(3)_C$, $q_{g_1} = q_{g_3} = 0$. At this order, λ dependence cannot appear either making $q_\lambda = 0$. So we are left with

$$\dot{M}_\nu = \frac{q_1}{2}\left[M_\nu\left(Y_\nu Y_\nu^\dagger\right) + \left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\right] + q_{g2} g_2^2 M_\nu . \quad (3.27)$$

Here, we keep the g_2^2 dependence to get the general form of $\dot{M}_\nu$ for right-handed neutrino extended models with $\mathrm{G_{LF}}$ flavor symmetry. For MLFV-ex, where the right-handed neutrinos are singlets under $\mathrm{SU}(2)_L$, we have $q_{g_2} = 0$. It should also be noted that if we use the universality of M_ν as an initial condition in Eq. (3.27), when $\mathrm{G_{LF}}$ is broken by the small background values of the spurion field Y_ν, the universality of the Majorana mass matrix is also broken as Y_ν has non-zero off-diagonal entries in general. However, the breaking is small and we can still consider $\mathrm{G_{LF}}$ as the flavor symmetry of the theory in the massless lepton limit and perform the spurion analysis.

Let us now consider the term containing $M_\nu^\dagger$, that involves the left-handed fields. Writing the indices explicitly, for a general M_ν matrix, we get that the mass term associated with the left-handed fields is $(M_\nu^*)_{\alpha\beta}$, instead of $(M_\nu)^{\alpha\beta}$ for the right-handed fields, and thus the allowed terms are

$$T'_\nu M_\nu = (1,1,8) \otimes (1,1,6) \ni (1,1,6) , \quad (3.28)$$

$$\mathrm{Tr}[Y_e^\dagger Y_e] M_\nu = (1,1,1) \otimes (1,1,6) = (1,1,6) , \quad (3.29)$$

$$\mathrm{Tr}[Y_\nu^\dagger Y_\nu] M_\nu = (1,1,1) \otimes (1,1,6) = (1,1,6) . \quad (3.30)$$

Hence after symmetrization the evolution equation of the right-handed neutrino mass has a Dirac structure and is given by

$$\dot{M}_\nu = \frac{q_1}{2}\left[\left(M_\nu\left(Y_\nu Y_\nu^\dagger\right) + \left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\right) P_R + \left(\left(Y_\nu Y_\nu^\dagger\right) M_\nu + M_\nu\left(Y_\nu Y_\nu^\dagger\right)^T\right) P_L\right] + q_{g_2} g_2^2 M_\nu . \quad (3.31)$$

Eq. (3.31) is the most general form of M_ν evolution, as obtained by loop diagram calculations in [9, 10].

D. RG evolution of the left-handed Majorana mass m_ν

At energy scales above M_R, the light left-handed neutrino mass m_ν is generated through the seesaw relation and hence the RG evolution of m_ν will be obtained through that of Y_ν

and M_ν, as given in Eqs. (3.20) and (3.27). Using the seesaw relation given in Eq. (2.9) and considering the fact that $(M_\nu^{-1})^{\alpha\gamma}(M_\nu)_{\gamma\beta} = \delta^\alpha{}_\beta$, we see that to get the RG evolution equation for $(m_\nu)^{ij}$, i, j being the $\mathrm{SU}(2)_{l_L}$ indices, one needs the evolution of $(M_\nu)_{\alpha\beta}$, i.e. the left-chiral projection of the RG evolution of M_ν, which can be read off from Eq. (3.31). Finally, the evolution equation for m_ν is given by

$$\dot{m}_\nu \;=\; m_\nu P + P^T m_\nu + p m_\nu \;, \tag{3.32}$$

where

$$P \;=\; b_1 Y_e^\dagger Y_e + \left(b_2 - \frac{q_1}{2}\right) Y_\nu^\dagger Y_\nu \;, \tag{3.33}$$

$$p \;=\; 2\left(b_T T + b_{g_1} g_1^2 + b_{g_2} g_2^2\right) - q_{g_2} g_2^2 \;. \tag{3.34}$$

Note that the RHS of the equation is symmetric under $\mathrm{SU}(3)_{l_L}$, as required. All of $b_{1,2}$, b_T, $b_{g_{1,2}}$ and q_1 are given below for the cases of Type-I and Type-III seesaw in SM and MSSM.

E. RG evolution at energies below M_R

To complete the discussion of RG evolution of the different quantities that are needed in order to have a complete description of all leptonic parameters at all energy scales, we now construct the RG evolution equations for $\mu < M_R$. In this regime the flavor symmetry is

$$\mathrm{G}'_{\mathrm{LF}} \equiv \mathrm{SU}(3)_{l_L} \otimes \mathrm{SU}(3)_{e_R} \;, \tag{3.35}$$

and the Yukawa coupling, $Y_e(\bar{3}, 3)$, and the left-handed Majorana mass, $m_\nu(6, 1)$, are the only spurion fields. The RG evolution equation for Y_e can be obtained following the procedure given in the last subsection to be

$$\dot{Y}_e \;=\; Y_e\left(a_1 Y_e^\dagger Y_e + a_T T' + a_{g_1} g_1^2 + a_{g_2} g_2^2\right) \;, \tag{3.36}$$

where

$$T' \equiv \mathrm{Tr}[Y_e^\dagger Y_e] + 3\mathrm{Tr}[Y_U^\dagger Y_U] + 3\mathrm{Tr}[Y_D^\dagger Y_D] \;, \tag{3.37}$$

and a_is are expected to be of $\mathcal{O}(1)$ as before.

In the low energy regime m_ν is an effective neutrino mass operator and its RG evolution is not given by Eq. (3.32). To determine the structure of the RG evolution equation for the left-handed Majorana mass m_ν, we proceed in the same way as in case of M_ν, keeping in mind the change in the chirality. Table I and the transformation rule in Eq. (3.22) can be used to determine the allowed combinations of m_ν and Y_e that can appear on the RHS of $\dot{m}_\nu$ and those are $m_\nu T_e$ and $m_\nu \mathrm{Tr}[Y_e^\dagger Y_e]$. Symmetrized over the $\mathrm{SU}(3)_{l_L}$ indices, the most general form of $\dot{m}_\nu$, keeping 1-loop spurion contributions, is

$$\begin{aligned}\dot{m}_\nu \;&=\; \frac{p}{2}\left(m_\nu T_e + (m_\nu T_e)^T\right) + p_e \mathrm{Tr}[Y_e^\dagger Y_e] \\ &+\; m_\nu\left(p_U \mathrm{Tr}[Y_U^\dagger Y_U] + p_D \mathrm{Tr}[Y_D^\dagger Y_D] + p_{g_1} g_1^2 + p_{g_2} g_2^2 + p_\lambda \lambda\right) \;,\end{aligned} \tag{3.38}$$

which can be simplified to

$$\dot{m}_\nu = \frac{p_1}{2}\left(m_\nu Y_e^\dagger Y_e + (Y_e^\dagger Y_e)^T m_\nu\right) + m_\nu(p_T T' + p_{g_1} g_1^2 + p_{g_2} g_2^2 + p_\lambda \lambda) , \tag{3.39}$$

where T' has been defined in Eq. (3.37). As before, we have considered the $\mathrm{SU}(3)_C$ charges of the quarks in fixing $p_{U,D}$ and writing T'. Here p_is are the $\mathcal{O}(1)$ numbers and we have used the fact that m_ν is symmetric under $\mathrm{SU}(3)_{l_L}$.

IV. RESULTS

To illustrate the RGEs obtained in Section III using spurion analysis, we compare the coefficients with the evolution equations obtained by exact calculations in four different models. These models are the extended SM and MSSM, where the right-handed neutrinos can be singlets (Type-I seesaw [9, 11, 12]) or triplets (Type-III seesaw [10]).

A. Right-handed neutrino extended SM

Let us first consider the case of the SM extended with three right-handed neutrinos. There can be only two possibilities: the first option is when the right-handed neutrinos are singlets under the gauge group which is known as Type-I seesaw. The other option, known as Type-III seesaw, is when the neutrinos are triplets under $\mathrm{SU}(2)_L$ and singlet under the remaining $\mathrm{SU}(3)_C \times U(1)_Y$. Note that for Type-II seesaw [13] as well as Inverse seesaw [14], the flavor group and the spurions present in the theory are not identical to the above cases and cannot be treated as a realization of the case discussed here.

In the general case of Type-I and Type-III seesaw, each of the right-handed neutrinos can be expressed as

$$\nu_R \equiv \sum_{a=1}^{N} \nu_R^a G^a , \tag{4.1}$$

with

$$\begin{aligned} G^a &\equiv \mathbb{I} , \quad N = 1 \quad \text{for Type-I seesaw} , \\ G^a &\equiv \sigma^a , \ N = 3 \quad \text{for Type-III seesaw} , \end{aligned} \tag{4.2}$$

where σ^a represent the Pauli matrices. Note that we work in three different spaces. The flavor index, $f = e, \mu, \tau$, is suppressed. There is also the internal SU(2) index of the Pauli matrices that we suppress here and in the rest of the paper. In the following we often get quantities that are universal in that index. Last, the explicit index a that runs from 1 to N.

With the above definition, we can write $r \equiv a_4/a_3$ in Eq. (3.13) as

$$r = \sum_{a=1}^{N} \epsilon^T G^a G^a \epsilon = (1, 3) , \tag{4.3}$$

where $\epsilon \equiv i\sigma^2$. The two numbers in the parenthesis are the values in Type-I and Type-III seesaws, and are universal in the SU(2) spaces.

The quantities that appear in the coefficients of $\dot{Y}_e$, $\dot{Y}_\nu$ and $\dot{M}_\nu$, and depend on the representation of the right-handed neutrinos are

$$\alpha_1 = \sum_{a=1}^{N} G^a G^a = (1,3) \, , \tag{4.4}$$

$$\alpha_2 = \mathrm{Tr}[G^a G^a] = (2,2) \, , \tag{4.5}$$

$$\alpha_3 = \sum_{a=1}^{N} G^{aT} \epsilon G^a \epsilon = (-1,3) \, , \tag{4.6}$$

$$\alpha_4 = \left(\epsilon^T G^a\right)^T \left(\epsilon^T G^a\right)^{-1} = (-1,1) \, , \tag{4.7}$$

$$\alpha_5 = \sum_{a,b=1}^{N} \left(i\varepsilon^{bac} G^{aT} \epsilon^T G^b\right) \left(\epsilon^T G^c\right)^{-1} = (0,-2) \, , \tag{4.8}$$

where ε^{bac} is the completely anti-symmetric tensor in SU(2) indices and no summation convention has been used.

Let us now discuss the origin of α_is. α_1 comes from the self-energy correction of l_L, while α_2 appears in the self-energy correction of ν_R. α_3 comes in the correction of the vertex containing Y_e, while α_4 is present in the correction of the Y_ν vertex. α_5 appears in the vertex correction of Y_ν because of $\mathrm{SU}(2)_L$ interactions. In the case of right-handed neutrino extended SM, self-energy, mass and vertex corrections contribute to the running of the Yukawa couplings $Y_{e,\nu}$. Hence, α_1 is expected to contribute to both $\dot{Y}_e$ and $\dot{Y}_\nu$, while $\dot{Y}_e$ should contain α_3 as well. α_4 and α_5 must appear in $\dot{Y}_\nu$. As already discussed, these quantities do not appear in $\dot{m}_\nu$ in the regime $\mu < M_R$, since the right-handed neutrinos are already decoupled.

Let us now consider the coefficients $a_{1,2}$, a_T and a_{g_1,g_2} arising in $\dot{Y}_e$ in Eq. (3.14). Collecting all the contributions, we get the coefficients in Eq. (3.14) to be [9, 10]

$$a_1 = \frac{3}{2} \, , \quad a_2 = \frac{\alpha_1}{2} + 2\alpha_3 \, , \quad a_T = 1 \, , \quad a_{g_1} = \left(-\frac{3}{4} - 3\right) \times \frac{3}{5} \, , \quad a_{g_2} = -3 \times \frac{3}{4} \, . \tag{4.9}$$

The first term in a_{g_1} arises through the self-energy correction of Higgs field ϕ, which also contributes to a_{g_2}. Here we have used GUT normalization for $\mathrm{U}(1)_Y$ charges and hence a factor of (3/5) comes with g_1^2. The coefficients appearing in the RG evolution equation of Y_ν in Eq. (3.20) can also be obtained in a similar way and we have [9, 10]

$$b_1 = \frac{1}{2} + 2\alpha_4 \, , \quad b_2 = \frac{1}{2}\left(\alpha_1 + \alpha_2\right) \, , \quad b_T = 1 \, , \quad b_{g_1} = -\frac{3}{4} \times \frac{3}{5} \, , \quad b_{g_2} = -3 \times \frac{3}{4} + 3\alpha_5 \, . \tag{4.10}$$

The values of a_i and b_i in Type-I and Type-III seesaw scenarios are tabulated in Table II. As can be seen from the table, for Type-I seesaw in the extended SM model the coefficients are

		SM		MSSM	
		Type-I	Type-III	Type-I	Type-III
$T(T_U)$	$r(r')$	1	3	1	3
$\dot{Y}_e$	a_1	$3/2$	$3/2$	3	3
	a_2	$-3/2$	$15/2$	1	3
	a_T	1	1	1	1
	a_{g_1}	$-9/4$	$-9/4$	$-9/5$	$-9/5$
	a_{g_2}	$-9/4$	$-9/4$	-3	-3
$\dot{Y}_\nu$	b_1	$-3/2$	$5/2$	1	1
	b_2	$3/2$	$5/2$	3	5
	b_T	1	1	1	1
	b_{g_1}	$-9/20$	$-9/20$	$-3/5$	$-3/5$
	b_{g_2}	$-9/4$	$-33/4$	-3	-7
$\dot{M}_\nu$	q_1	2	2	4	4
	q_{g_2}	0	-12	0	-8
$\dot{m}_\nu$ $(\mu < M_R)$	p_1	-3		-3	
	p_T	2		2	
	p_{g_1}	0		$-6/5$	
	p_{g_2}	-3		-6	
	p_λ	1		0	

TABLE II: Coefficients appearing in the RG evolution of Y_e, Y_ν, M_ν and m_ν in the SM and MSSM, in case of Type-I and Type-III seesaw [9–12]. For the extended MSSM, a_T is the coefficient of T_D, while b_T and p_T are of T_U and T'_U, respectively.

$\mathcal{O}(1)$ numbers, as expected. In the case of the Type-III seesaw, we see that there are numbers which are larger than $\mathcal{O}(1)$, for example a_2 and b_{g_2}. Let us now try to understand the origin of these large numbers. The largest contribution to a_2 comes from the α_3 in Eq. (4.9), which arises through the vertex correction due to right-handed triplets and a factor of three is expected. Thus, the relevant number which we expect to be of $\mathcal{O}(1)$ is $(a_2/3)$. Moreover, the right-handed neutrino triplets have interactions with the $\mathrm{SU}(2)_L$ gauge bosons over the singlets, and so we expect b_{g_2} in the Type-III case to have a factor of six over b_{g_2} in Type-I.

Let us now discuss the coefficients q_1 and q_{g_2} appearing in the running of M_ν. The coefficients are given by

$$q_1 = \alpha_2 \,, \qquad q_{g_2} = (0, -12) \,, \tag{4.11}$$

where α_2 is defined in Eq. (4.5) and is of $\mathcal{O}(1)$. For Type-I seesaw, the right-handed neutrinos are singlets of $\mathrm{SU}(2)_L$ and so $q_{g_2} = 0$, while for Type-III seesaw one gets by exact calculations

[10] $q_{g_2} = -12$ and $(q_{g_2}/6)$ is of $\mathcal{O}(1)$, as discussed earlier.

Last, we consider the evolution of the effective left-handed Majorana neutrino mass m_ν in the energy scales $\mu < M_R$. In this energy regime, the evolution equations are the same for all the different seesaws, since we are considering an effective theory. However they will depend on the underlying theory, which is the SM in this case. The values of different p_is are given in Table II and are of $\mathcal{O}(1)$ as anticipated.

Note that explicit 1-loop calculations show that $p_{g_1} = 0$. We were unable to find an explanation based on symmetry considerations and hence we think it is accidental. We expect g_1^2 dependent terms to emerge at 2-loop.

B. Right-handed neutrino extended MSSM

We now consider the case of the MSSM extended by three right-handed neutrinos. Our formalism is applicable in this case as well, since the flavor structure of the MSSM is identical to that of the SM. But the Higgs sector of MSSM is different. One of the Higgses, H_U, couples to leptons through the Yukawa coupling Y_U to give rise to the up-type lepton masses, while the other Higgs, H_D, is responsible for the down-type lepton masses through the Yukawa coupling Y_D. Hence there are two types of trace terms. The first is T_U which is a combination of $\mathrm{Tr}[Y_\nu^\dagger Y_\nu]$ and $\mathrm{Tr}[Y_U^\dagger Y_U]$. The other one is T_D, a combination of $\mathrm{Tr}[Y_e^\dagger Y_e]$ and $\mathrm{Tr}[Y_D^\dagger Y_D]$. We define the trace terms as

$$T_U = r'\mathrm{Tr}[Y_\nu^\dagger Y_\nu] + 3\mathrm{Tr}[Y_U^\dagger Y_U] \ , \tag{4.12}$$

$$T_D = \mathrm{Tr}[Y_e^\dagger Y_e] + 3\mathrm{Tr}[Y_D^\dagger Y_D] \ , \tag{4.13}$$

where

$$r' \equiv \sum_{i=1}^{N} \left(\epsilon G^i\right)^* \left(\epsilon G^i\right)^T = (1,3) \tag{4.14}$$

is a quantity, similar to r defined in Eq. (4.3) in the SM, that depends on the transformation of the right-handed neutrinos under the gauge group. The two numbers in the parenthesis are the values in Type-I and Type-III seesaw scenarios. As before, r' is universal in SU(2) spaces and we write down the universality constant only.

Let us now define the quantities that contribute to the evolution of Y_e, Y_ν and M_ν in Type-I and Type-III seesaws and depend on the gauge group representations of the right-handed neutrinos:

$$\alpha_1' = \sum_{a=1}^{N} (\epsilon G^a)^\dagger (\epsilon G^a) = (1,3) \ , \tag{4.15}$$

$$\alpha_2' = \mathrm{Tr}[(\epsilon G^a)^\dagger (\epsilon G^a)] = (2,2) \ , \tag{4.16}$$

$$C_2 = (0,2) \ . \tag{4.17}$$

C_2 is the quadratic Casimir for the irreducible representation $\mathcal{R}$ of $\mathrm{SU}(2)_L$ in which the right-handed neutrinos ν_R^i reside. For Type-I seesaw $C_2 = 0$, while for Type-III seesaw the right-handed fields are in the adjoint representation of $\mathrm{SU}(2)_L$ and hence $C_2 = 2$. RG evolution of Yukawas and masses in Type-III seesaw with MSSM as the underlying theory has not been computed before We give some details of the calculation in Appendix A.

Let us now write down the coefficients involved in $\dot{Y}_e$ in Eq. (3.14).

$$a_1 = 3 \,, \quad a_2 = \alpha_1' \,, \quad a_T = 1 \,, \quad a_{g_1} = -3 \times \frac{3}{5} \,, \quad a_{g_2} = -3 \,. \tag{4.18}$$

We see that in the MSSM, as in the case of the SM, only a_2, the coefficient of $Y_\nu^\dagger Y_\nu$, depends on whether the seesaw is Type-I or Type-III. For the case of $\dot{Y}_\nu$, the coefficients appearing in Eq. (3.20) are

$$b_1 = 1 \,, \quad b_2 = \alpha_1' + \alpha_2' \,, \quad b_T = 1 \,, \quad b_{g_1} = -\frac{3}{5} \,, \quad b_{g_2} = -3 - 2C_2 \,. \tag{4.19}$$

Comparing the expressions of b_1 in the SM and the MSSM, in Eqs. (4.10) and (4.19), we see that in the SM b_1 receives a contribution that depends on the right-handed neutrinos, which is absent in MSSM. This is to be attributed to the non-renormalization theorem due to which only the wavefunction renormalizations are responsible for the RG evolution of the quantities in MSSM and the mass and vertex corrections do not contribute. The absence of any vertex renormalization contribution makes b_1 independent of the right-handed neutrino fields in MSSM. The values of a_i and b_i in the two seesaw types are given in Table II.

From Table II it is seen that for Type-I seesaw scenario, all the numbers are of $\mathcal{O}(1)$ and consistent with prediction from spurion analysis. However, for Type-III seesaw both b_2 and b_{g_2} are large numbers, the large contribution emerging from the wavefunction renormalization of the superfields l and ν respectively.

Next, we move to the case of the right-handed Majorana mass M_ν. The coefficients are

$$q_1 = 2\alpha_2' \,, \quad q_{g_2} = -4C_2 \,, \tag{4.20}$$

where α_2' and C_2 have already been defined in Eqs. (4.16) and (4.17) respectively. Values of q_1 and q_{g_2} in the two types of seesaw scenarios are listed in Table II. As expected, $q_{g_2} = 0$ and q_1 is of $\mathcal{O}(1)$ in Type-I seesaw, while for Type-III seesaw q_1 and $(q_{g_2}/6)$ are $\mathcal{O}(1)$ numbers.

For energies $\mu < M_R$, evolution of the left-handed neutrino mass m_ν is the same in both Type-I and Type-III seesaws and the values of the coefficients [11, 12] are quoted in Table II. Note that the accidental cancellation seen in the SM case, $p_{g_1} = 0$, does not happen in the MSSM. The trace term appearing in this case is $T' \to T_U' = \mathrm{Tr}[3Y_U^\dagger Y_U]$, since in the high energy theory only H_U interacts with ν. The Higgs self-coupling term with coefficient p_λ does not exist in this scenario.

The above comparison shows that the method of spurion analysis gives the form of the RG evolution equations. Of course, working in a generic effective field theory we never

expect to get the exact values of the $\mathcal{O}(1)$ numbers, which depend on the specific details of the model. One can use this same technique to get the evolution equations at second order. Calculation of evolution equations at 2-loop and comparison with the existing results obtained by loop calculations is given in Appendix B.

V. BREAKING DEGENERACY OF M_ν AND LEPTOGENESIS

In this section, we study effects related to the breaking of the universality of M_ν. This breaking is important in the context of leptogenesis. It has been studied in detail in [3] where the mass degeneracy is removed by appropriate combinations of spurions transforming as $(1,1,6)$ under $\mathrm{G_{LF}}$. Here we compare their results of explicit breaking with the effects generated through RG evolution.

We start with the case of degeneracy breaking by RG evolution. For this purpose, writing down the evolution equation for a component of M_ν from Eq. (3.27) we get

$$\left(\dot{M}_\nu\right)_{ij} = \frac{q_1}{2}\left[(M_\nu)_{ik}\left(Y_\nu Y_\nu^\dagger\right)_{kj} + \left(Y_\nu Y_\nu^\dagger\right)_{ki}(M_\nu)_{kj}\right] + q_{g_2} g_2^2 (M_\nu)_{ij} \ . \tag{5.1}$$

Using universal-mass initial condition, $(M_\nu)_{ij} = M_R \delta_{ij}$, one gets the final eigen-values of M_ν after RG running to be non-degenerate. The specific value of breaking depends on the values of q_1, q_{g_2} as well as the RG evolution of the spurion field Y_ν and its background value, and thus on the underlying theory considered.

Next, we study degeneracy breaking at the high scale using spurion techniques. To the lowest order in the spurion fields $Y_{e,\nu}$, the final Majorana mass matrix M_ν^F is written as

$$M_\nu^F = M_\nu + \sum_n c_n \delta M_\nu^{(n)} \ , \tag{5.2}$$

where $M_\nu = M_R \mathbb{I}$ is the universal mass matrix given in Eq. (2.3) and

$$\begin{aligned}
\delta M_\nu^{(11)} &= M_R\left(Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger)^T\right) \ , \\
\delta M_\nu^{(21)} &= M_R\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T\right) \ , \\
\delta M_\nu^{(22)} &= M_R\left(Y_\nu Y_\nu^\dagger (Y_\nu Y_\nu^\dagger)^T\right) \ , \\
\delta M_\nu^{(23)} &= M_R\left((Y_\nu Y_\nu^\dagger)^T Y_\nu Y_\nu^\dagger\right) \ , \\
\delta M_\nu^{(24)} &= M_R\left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger + (Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger)^T\right) \ ,
\end{aligned} \tag{5.3}$$

considering terms containing up to four spurions. As discussed in [3], values of c_n depends on dynamical properties: if the Yukawa corrections are generated within a perturbative regime, as is the case for RG evolution, c_n decreases according to the power of Yukawa matrices, for example, in a standard loop-expansion one should have $c_{11} \sim g_{\rm eff}^2/(4\pi)^2$ and then $c_{2i} \sim c_{11}^2$ and so on. One cannot exclude a priori a strong-interaction regime where $c_n \sim \mathcal{O}(1)$, for

all n. But even in the case of strong-interaction, the series in Eq. (5.2) is expected to be dominated by the first few terms as the background values of the spurions $Y_{e,\nu}$ are small. In this paper, we consider the perturbative regime of explicit breaking only.

In Ref. [3] it is shown that the amount of mass degeneracy breaking is important in the context of leptogenesis. In the rest of this section we consider the two sources of breaking and study the pattern of mass universality breaking and its effect on leptogenesis. We briefly describe the parametrization of the Yukawa Y_ν following [3]. We choose to work in the basis where Y_e is diagonal. Then the neutrino mass matrix is given as

$$m_\nu = U^*_{\text{PMNS}} m_\nu^{\text{diag}} U^\dagger_{\text{PMNS}} \,, \tag{5.4}$$

where

$$m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) \tag{5.5}$$

and U_{PMNS} is the unitary matrix that diagonalizes m_ν. In this basis, the most general form of Y_ν is given by the Casas-Ibarra parametrization [15]:

$$Y_\nu \;=\; \frac{1}{v} M_\nu^{1/2} R \big(m_\nu^{\text{diag}}\big)^{1/2} U^\dagger_{\text{PMNS}} = \frac{\sqrt{M_R}}{v} R \big(m_\nu^{\text{diag}}\big)^{1/2} U^\dagger_{\text{PMNS}} \,, \tag{5.6}$$

where R is a complex orthogonal matrix parametrized by six real quantities. We write $R = OH$, where O is a real orthogonal matrix and H is complex orthogonal hermitian matrix and thus each O and H contains three real parameters. Since $O \in O(3)_{\nu_R}$, and $O(3)_{\nu_R}$ is a symmetry of the theory independent of any assumption on CP properties, we can choose $O \equiv \mathbb{I}$ to get $R = H$. Thus finally

$$Y_\nu \;=\; \frac{\sqrt{M_R}}{v} H \big(m_\nu^{\text{diag}}\big)^{1/2} U^\dagger_{\text{PMNS}} \,. \tag{5.7}$$

In the CP conserving limit, $H = \mathbb{I}$. The CP violating nature of H is clear in the following parametrization [16]:

$$H = e^{i\Phi} = \mathbb{I} - \frac{\cosh\rho - 1}{\rho^2}\Phi^2 + i\frac{\sinh\rho}{\rho}\Phi \,, \tag{5.8}$$

where

$$\rho = \sqrt{\varphi_1^2 + \varphi_2^2 + \varphi_3^2} \,, \qquad \text{and} \qquad \Phi \;=\; \begin{pmatrix} 0 & \varphi_1 & \varphi_2 \\ -\varphi_1 & 0 & \varphi_3 \\ -\varphi_2 & -\varphi_3 & 0 \end{pmatrix} . \tag{5.9}$$

Let us now proceed to the numeric example. In the generic case of [3], the breaking depends on the choice of c_ns, while in case of RG evolution we need to specify the underlying theory (for example SM or MSSM and also Type-I or Type-III). In both cases, the mass-splitting of the right-handed neutrinos depends on Φ as well as the neutrino masses and

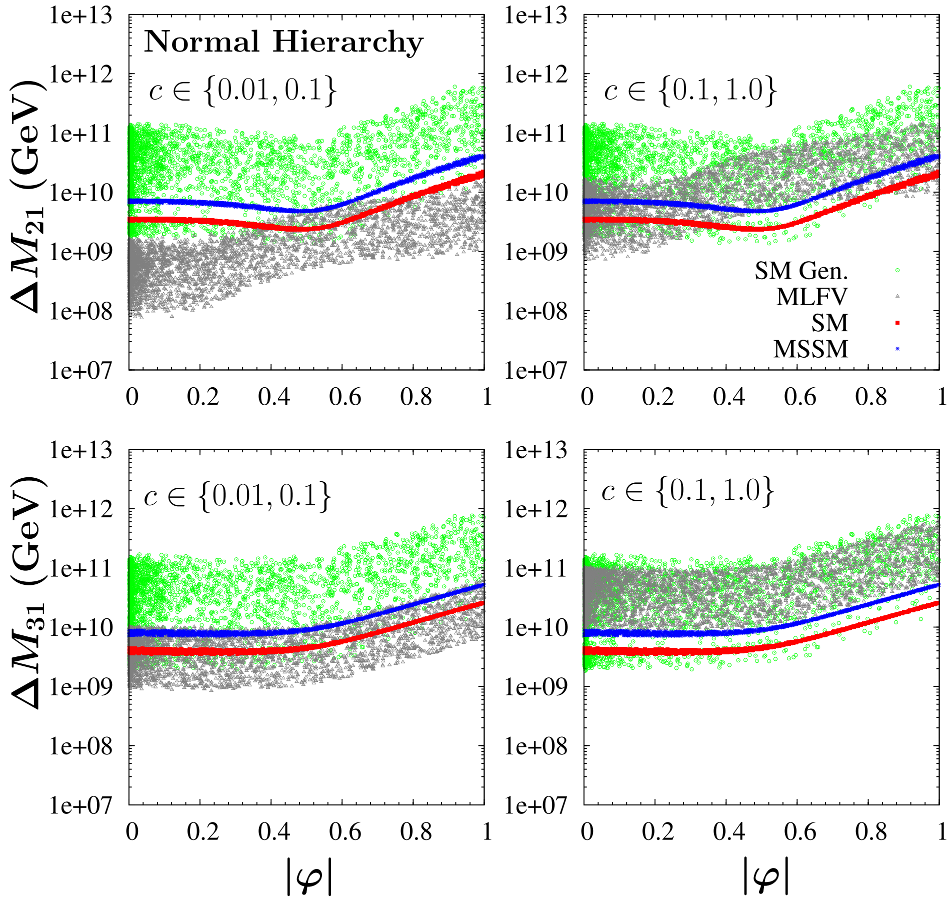

FIG. 2: Majorana mass splittings as a function of $|\varphi|$ for normal neutrino mass hierarchy. We have defined $\Delta M_{i1} = M_i - M_1$. The green (light gray in black and white) dots show 'SM Gen.', and dark gray dots are for 'MLFV'. The red (lower) and blue (upper) dotted lines correspond to SM and MSSM respectively.

mixing parameters through Y_ν. For the purpose of illustration we choose $M_R = 10^{13}$ GeV, and $\varphi_1 = \varphi_2 = \varphi_3 = \varphi$, and then consider the range $10^{-3} \leq |\varphi| \leq 1$. The neutrino mass-squared differences are set to the central experimental values: $|\Delta m^2_{32}| = |m_3^2 - m_2^2| = 2.4 \times 10^{-3}$ eV^2 and $\Delta m^2_{21} = m_2^2 - m_1^2 = 7.65 \times 10^{-5}$ eV^2. The lightest neutrino mass is chosen to be in the range $\{10^{-4}, 10^{-2}\}$ eV. The mixing angles have been fixed to tribimaximal values. Finally, points satisfying $|\,(Y_\nu)_{ij}\,| \leq 1$ are considered. For the MSSM, we take $\tan\beta = 20$.

To illustrate the mass-splitting generated through RG evolution, we consider the case of Type-I seesaw and show the results when the theory is extended SM, extended MSSM and also any generic theory with the same underlying symmetry as extended SM (referred to as 'SM Gen.'). All these cases together are referred to as 'Type-I RG'. In case of 'SM Gen.', we choose the coefficients appearing in the evolution of Y_e, Y_ν and M_ν, given in Eqs. (3.14),

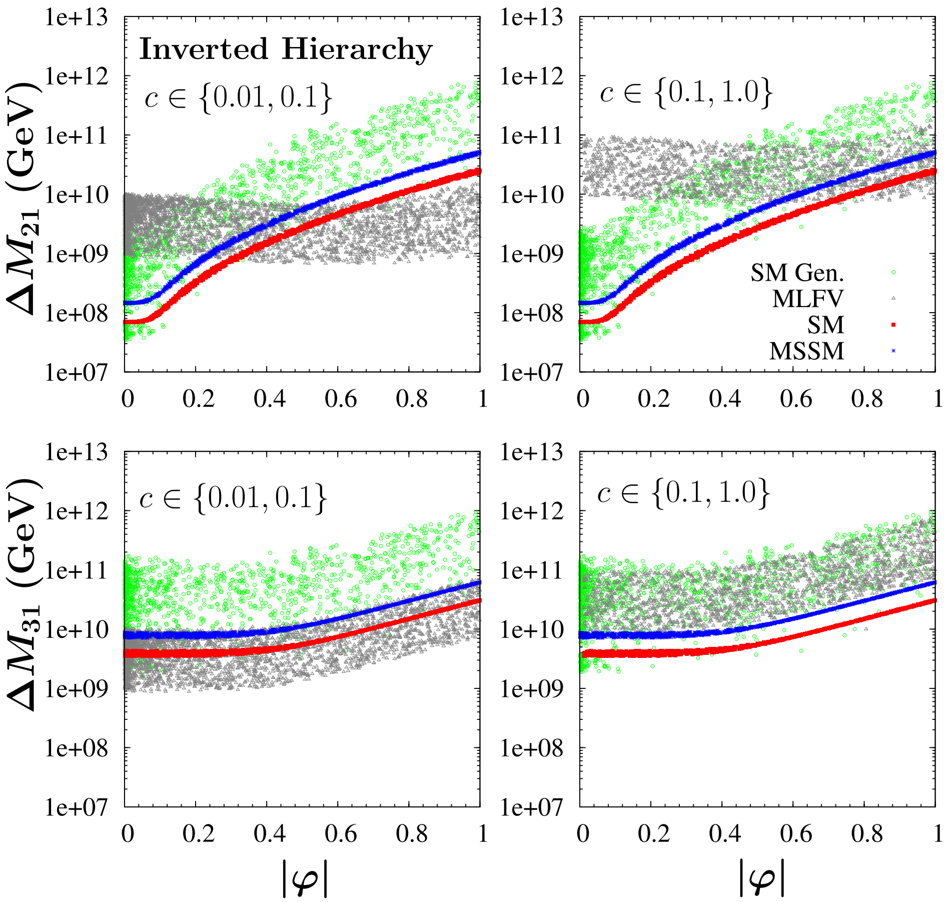

FIG. 3: Majorana mass splittings as a function of $|\varphi|$ for an inverted hierarchy of neutrino masses. The green (light gray in black and white) dots show 'SM Gen.', and dark gray dots are for 'MLFV'. The red (lower) and blue (upper) dotted lines correspond to SM and MSSM respectively.

(3.20) and (3.27) respectively, as

$$|a_{1,2}|, |b_{1,2}|, q_1, -a_{g_1,g_2}, -b_{g_1,g_2} \in \{0.5, 4\} \ , \qquad a_T = b_T = 1 \ , \qquad q_{g_2} = 0 \ . \tag{5.10}$$

For 'Type-I RG', the high scale is chosen to be $\mu_0 = 10^{16}$ GeV, while the value of the mass-splitting is evaluated at $\mu = M_R$. For the general MLFV scenario [3] (referred to as 'MLFV'), we consider the case when

$$c_{11} = c \qquad \text{and} \qquad c_{21} = c_{24} = c^2 \ , \tag{5.11}$$

with all other c_ns set to zero. The value of c is varied over a few orders of magnitude, $c \in \{10^{-2}, 1\}$, as can be seen in Figs. 2 and 3. In the 'MLFV' scenario, the mass-splitting does not depend on the energy scale.

Fig. 2 shows the plots for normal neutrino hierarchy ($\Delta m^2_{32} > 0$), while Fig. 3 shows that for the inverted case ($\Delta m^2_{32} < 0$). From the figures one can make the following observations:

- For both the cases of 'MLFV' and 'Type-I RG', the nature of variation of $\Delta M_{31} = M_3 - M_1$ with $|\varphi|$ is the same, for the whole range of $|\varphi| \in \{0.001, 1.0\}$, with either neutrino mass hierarchy. The generic variation trends are different for $\Delta M_{21} = M_2 - M_1$.

- In case of 'Type-I RG', for inverted hierarchy ΔM_{21} varies about two orders of magnitude as $|\varphi|$ is varied in the range $0.1 - 1.0$. For normal hierarchy, the variation is small for $|\varphi| \lesssim 0.5$. For 'MLFV', variation of ΔM_{21} is quite small for inverted hierarchy.

- There is an overlap of ΔM_{21} generated in 'MLFV' for $c \in \{0.01, 0.1\}$ with that in 'SM Gen.' for $|\varphi| > 0.3(0.15)$ with normal(inverted) hierarchy. For higher c values, $c \in \{0.1, 1.0\}$, the 'MLFV' can resemble the RG effect for the whole range of $|\varphi|$ for normal hierarchy, while for inverted hierarchy the same is accomplished for $|\varphi| > 0.2$.

- ΔM_{31} generated in 'MLFV' overlaps that in 'SM Gen.' for the whole range of $|\varphi|$ with both the hierarchies and for all $c \in \{0.001, 1.0\}$.

The above example shows a consistent treatment of the splitting that include both the generic splittings from spurion technique and the RG evolution. The result obtained in the case of a general splitting with spurions is different from what we get when RG effects are included. However, there is an overlap for some region of the parameter space.

Next, we discuss the effect of including RG evolution on leptogenesis, and compare it to the result obtained with the generic splitting [3]. The baryon asymmetry η_B can be expressed as

$$\eta_B = 9.6 \times 10^{-3} \sum_i \epsilon_i d_i \ , \tag{5.12}$$

where d_i are the washout factors, and the ϵ_i are the CP asymmetries defined as [17–19]

$$\epsilon_i = \frac{\sum_k \left[\Gamma(\nu_R^i \to l_k \phi^*) - \Gamma(\nu_R^i \to \bar{l}_k \phi)\right]}{\sum_k \left[\Gamma(\nu_R^i \to l_k \phi^*) + \Gamma(\nu_R^i \to \bar{l}_k \phi)\right]} \ . \tag{5.13}$$

To determine d_i, we consider the strong washout regime and use the same approximations as in [3, 20].

Values of η_B obtained as a function of $|\varphi|$ is shown in Fig. 4. The black (dashed) horizontal line shows the current experimental value of the baryon asymmetry [21]

$$\eta_B = (6.23 \pm 0.17) \times 10^{-10} \ , \tag{5.14}$$

at 1σ. It can be seen from Fig. 4 that in case of generic mass splitting with spurion techniques [3] the correct value of η_B can be achieved for $0.1 \lesssim |\varphi| \lesssim 0.4$, for the given choice of other parameters, with both the neutrino hierarchies and $c \in \{0.01, 0.1\}$. For other values of $|\varphi|$, the baryon asymmetry is lower than the current experimental value. For higher c values, $c \in \{0.1, 1.0\}$, the correct η_B is obtained for a small region around $|\varphi| \sim 0.1$ and

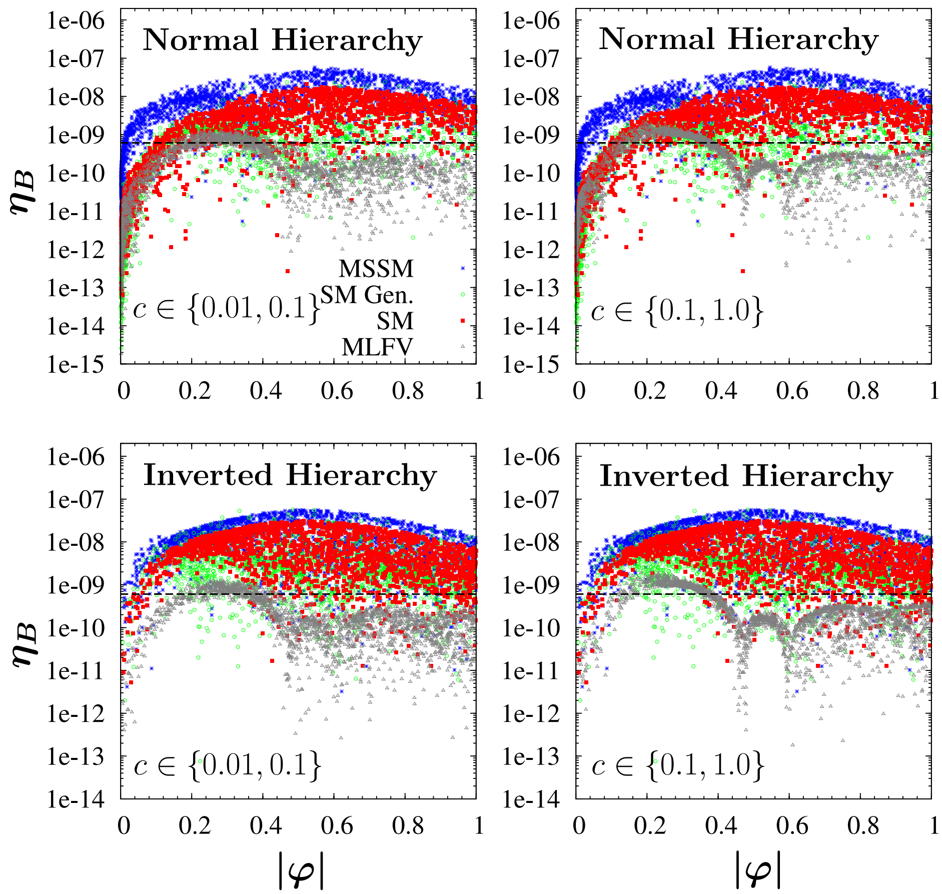

FIG. 4: Baryon asymmetry of the Universe, η_B, as a function of $|\varphi|$, for 'Type-I RG' and 'MLFV'. The black (dashed) horizontal line shows the current experimental values of η_B at 1σ.

$|\varphi| \sim 0.4$. However, if one considers 'Type-I RG', the correct baryon asymmetry is achieved for the whole $|\varphi|$ range and for both hierarchies. The results obtained in the two cases are different, with a small overlap in the allowed parameter space. Hence, while relating the low energy effects with the high energy phenomena, one must include the complete RG evolution of parameters, rather than considering a generic mass splitting to mimic the effect.

VI. CONCLUSION

Neutrino physics provides a window to the physics of very high scale. In order to learn about high energy physics, one need to use RGEs to connect the low and high energy scales. In this paper, we study models of MLFV and write the RGEs in terms of spurions that capture the whole effect. It is only the coefficient of each term that varies between models.

Our results serve as a check on the existing calculations. For example, we find that both in

the SM and MSSM, the difference between the right-handed neutrino representations enters only in one term, when we consider the evolution of the Yukawa matrix Y_e. For the purpose of illustration of our results, we have also computed the RGEs of Yukawas and masses in case of MSSM Type-III seesaw scenario, for the first time. If needed, this spurion analysis method to determine the RG evolution can be extended to two loop order, as has been done here, in which case we can check where the difference between Type-I and Type-III models resides. Our results can also be extended to other models. For example, in Type-II seesaw and Inverse seesaw, we have more sources of lepton flavor breaking. We can include them in the analysis in order to get more insight about where the running effects are coming from.

One implication of our results has to do with leptogenesis. Degenerate right-handed neutrinos cannot give the required baryon asymmetry of the Universe. Thus, they must be split. The splitting can be accomplished in two ways: explicitly with allowed spurion combinations from symmetry consideration, as is done in [3], or by considering RG evolution of different parameters consistently. We show that the effect of RG running can significantly change the allowed region of parameter space for successful leptogenesis compared to the explicit breaking, and hence should be taken into account.

Acknowledgments

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Appendix A: Calculation of RG evolution in MSSM Type-III seesaw

In this section we consider the MSSM extended by the addition of three right-handed triplet superfields ν. This is the only model out of the four we considered where explicit calculation does not exist in the literature, and thus we present it here.

The Yukawa part of the superpotential is given by

$$\begin{aligned}\mathcal{W}_{\text{Yukawa}} \;=\;& (Y_\nu)_{gf}\,\nu^{Cg} H_U l^f + (Y_e)_{gf}\,e^{Cg} H_D l^f \\ +\;& (Y_U)_{gf}\,u^{Cg} H_U Q^f + (Y_D)_{gf}\,d^{Cg} H_D Q^f_b\;,\end{aligned} \tag{A1}$$

where the first line corresponds to the Yukawa interactions for the lepton superfields, while the second line shows the Yukawa interactions for the quark superfields. The superfields e, u and d contain the $\mathrm{SU}(2)_L$-singlet charged leptons, down-type quarks and up-type quarks, while l and Q contain the $\mathrm{SU}(2)_L$ lepton and quark doublets, respectively. Superpotential

corresponding to the Majorana mass term for triplet neutrino superfields is

$$\mathcal{W}_{\rm Maj} \;=\; \frac{1}{2}\nu^{Cg}\,(M_\nu)_{gf}\,\nu^{Cf}\;. \tag{A2}$$

$\mathcal{W}_{\rm Maj}$ is important for the seesaw mechanism, but it does not take part in the RG evolution of different quantities.

1. Wavefunction renormalization constants

Let us consider a general supersymmetric gauge theory containing N_Φ superfields $\Phi^{(i)}$ that transform under the irreducible representations $\mathcal{R}_1^{(i)}\times\cdots\times\mathcal{R}_K^{(i)}$ of the gauge group $G_1\otimes\cdots\otimes G_K$. The renormalizable part of the superpotential is given as

$$\mathcal{W}_{\rm renorm} \;=\; \frac{1}{6}\sum_{i,j,k=1}^{N_\Phi}\lambda_{(ijk)}\Phi^{(i)}\Phi^{(j)}\Phi^{(k)}\;, \tag{A3}$$

where (ijk) implies symmetrization over the indices. Due to the non-renormalization theorem, the RG evolution equations for different operators of the superpotential are governed only by the wavefunction renormalization constants for the superfields $\Phi^{(i)}$, given as

$$Z_{ij} = \mathbb{I}_{ij} + \delta Z_{ij}\;. \tag{A4}$$

The bare and renormalized superfields, $\Phi_{\rm B}^{(i)}$ and $\Phi^{(i)}$, are then related as

$$\Phi_{\rm B}^{(i)} = \sum_{j=1}^{N_\Phi} Z_{ij}^{\frac{1}{2}}\,\Phi^{(j)}\;. \tag{A5}$$

Using dimensional regularization *via* dimensional reduction, the wavefunction renormalization constants, in $d=4-\varepsilon$ dimensions, at 1-loop are obtained as [22, 23]

$$\delta Z_{ij}^{(1)} \;=\; -\frac{1}{16\pi^2}\frac{1}{\varepsilon}\left[\sum_{k,l=1}^{N_\Phi}\lambda^*_{ikl}\lambda_{jkl} - 4\sum_{n=1}^{K} g_n^2 C_2(\mathcal{R}_n^{(i)})\delta_{ij}\right]\;, \tag{A6}$$

where $C_2(\mathcal{R}_n^{(i)})$ is the quadratic Casimir for the representation $\mathcal{R}_n^{(i)}$ of the gauge group G_n.

Comparing the superpotentials in Eqs. (A3) and (A1), and using Eq. (A6), we get the $1/\varepsilon$ coefficients of the wavefunction renormalization constants, for different lepton and Higgs

superfields, to be

$$- \ (4\pi)^2\, \delta Z_l = 2 Y_e^\dagger Y_e + 2 \left(\sum_a (\epsilon G^a)^\dagger\, (\epsilon G^a) \right) Y_\nu^\dagger Y_\nu - \frac{3}{5} g_1^2 - 3 g_2^2 \ , \tag{A7}$$

$$- \ (4\pi)^2\, \delta Z_{e^C} = 4 Y_e^* Y_e^T - \frac{12}{5} g_1^2 \ , \tag{A8}$$

$$- \ (4\pi)^2\, \delta Z_{\nu^C} = 2 \ \mathrm{Tr}[(\epsilon G^a)^\dagger\, \epsilon G^a]\ Y_\nu^* Y_\nu^T - 4\ C_2(\mathcal{R}_{\mathrm{SU}(2)_L})\ g_2^2 \ , \tag{A9}$$

$$- \ (4\pi)^2\, \delta Z_{H^U} = 2 \left(\sum_a (\epsilon G^a)^*\, (\epsilon G^a)^T \right) \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + 6 \mathrm{Tr}[Y_U^\dagger Y_U] - \frac{3}{5} g_1^2 - 3 g_2^2 \ , \tag{A10}$$

$$- \ (4\pi)^2\, \delta Z_{H^D} = 2 \mathrm{Tr}[Y_e^\dagger Y_e] + 6 \mathrm{Tr}[Y_D^\dagger Y_D] - \frac{3}{5} g_1^2 - 3 g_2^2 \ . \tag{A11}$$

It must be noted that the wavefunction renormalization constants, given in Eqs. (A7) – (A9), are in general forms applicable to both Type-I and Type-III seesaw when we use appropriate forms of G^a, as given in Eq. (4.2). Thus the quantities, which depend on the transformation properties of the right-handed neutrino superfields, are

$$r' \ = \ \sum_a (\epsilon G^a)^*\, (\epsilon G^a)^T = (1, 3) \ , \tag{A12}$$

$$\alpha_1' \ = \ \sum_a (\epsilon G^a)^\dagger\, (\epsilon G^a) = (1, 3) \ , \tag{A13}$$

$$\alpha_2' \ = \ \mathrm{Tr} \left[(\epsilon G^a)^\dagger\, \epsilon G^a \right] = (2, 2) \ . \tag{A14}$$

Here the numbers in the parenthesis are the values in Type-I and Type-III seesaw scenarios, and are universal in the SU(2) space, as defined in Eqs. (4.14) – (4.16). We do not use any summation convention here. $C_2(\mathcal{R}_{\mathrm{SU}(2)_L})$ in Eq. (A9) is the quadratic Casimir for the superfield ν under $\mathrm{SU}(2)_L$ and hence, as given in Eq. (4.17), $C_2(\mathcal{R}_{\mathrm{SU}(2)_L}) = 0$ for Type-I seesaw, and $C_2(\mathcal{R}_{\mathrm{SU}(2)_L}) = 2$ for Type-III seesaw. In Section IV and in the remainder of the appendix we use $C_2 \equiv C_2(\mathcal{R}_{\mathrm{SU}(2)_L})$.

2. Calculation of RG evolution equations

Let us now compute the β-functions. The RG evolution of Y_e is given by

$$\mu \frac{dY_e}{d\mu} \ = \ -\frac{1}{2} \left(Y_e \delta Z_l + Y_e \delta Z_{H_D} + \delta Z_{e^C}^* Y_e \right) \ , \tag{A15}$$

which reduces to

$$\begin{aligned} \dot{Y}_e \ &= \ Y_e \left[3 Y_e^\dagger Y_e + \alpha_1' Y_\nu^\dagger Y_\nu + \Big(\mathrm{Tr}[Y_e^\dagger Y_e] + 3 \mathrm{Tr}[Y_D^\dagger Y_D] \Big) - \frac{9}{5} g_1^2 - 3 g_2^2 \right] \\ &= \ Y_e \left[3 Y_e^\dagger Y_e + \alpha_1' Y_\nu^\dagger Y_\nu + T_D - \frac{9}{5} g_1^2 - 3 g_2^2 \right] \ , \end{aligned} \tag{A16}$$

where

$$T_D = \mathrm{Tr}[Y_e^\dagger Y_e] + 3\mathrm{Tr}[Y_D^\dagger Y_D]\ . \tag{A17}$$

Similarly, the evolution equation for Y_ν is given by

$$\begin{aligned}\dot{Y}_\nu \ &= \ Y_\nu \left[Y_e^\dagger Y_e + (\alpha_1' + \alpha_2')\, Y_\nu^\dagger Y_\nu + \left(r' \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + 3\mathrm{Tr}[Y_U^\dagger Y_U] \right) - \frac{3}{5} g_1^2 - (3 + 2C_2)\, g_2^2 \right] \\ &= \ Y_\nu \left[Y_e^\dagger Y_e + (\alpha_1' + \alpha_2')\, Y_\nu^\dagger Y_\nu + T_U - \frac{3}{5} g_1^2 - (3 + 2C_2)\, g_2^2 \right] \end{aligned} \tag{A18}$$

where

$$T_U = r' \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + 3\mathrm{Tr}[Y_U^\dagger Y_U]\ . \tag{A19}$$

The evolution equation of the right-handed neutrino mass M_ν is given by

$$\mu \frac{dM_\nu}{d\mu} \ = \ -\frac{1}{2} \left(\delta Z_{\nu^C}^T M_\nu + M_\nu \delta Z_{\nu^C} \right)\ , \tag{A20}$$

which reduces to

$$\dot{M}_\nu \ = \ \alpha_2' \left[\left(Y_\nu Y_\nu^\dagger \right) M_\nu + M_\nu \left(Y_\nu Y_\nu^\dagger \right)^T \right] - 4 C_2 g_2^2 M_\nu\ . \tag{A21}$$

Appendix B: RG evolution equations at 2-loop

In Section III of the main part of the paper, we have considered the first order contribution of the spurion fields. Here, we study the second order terms in the RGEs of the Yukawas and the masses using the same technique.

1. 2-loop running of Y_e

In this section, we consider the evolution of Y_e. The new contributions at 2-loop will consist of five spurion fields transforming as $(\bar{3}, 3, 1)$ under $\mathrm{G_{LF}}$. Any combination of three spurion fields with $(\bar{3}, 3, 1)$ and two other couplings in the theory transforming trivially under $\mathrm{G_{LF}}$ is also a valid term at this order. There must also be terms proportional to a single spurion field and four other couplings.

Using Table I, the SU(3)-algebra

$$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1\ , \tag{B1}$$

and those given in Eq. (3.2), and the transformation properties

$$\mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] = (1,1,1)\ , \quad \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] = (1,1,1)\ , \quad \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu] = (1,1,1)\ , \tag{B2}$$

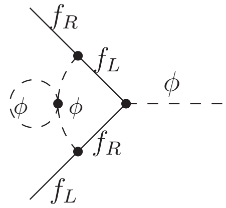

FIG. 5: Example of diagram contributing terms proportional to d^λ_{12}, d^λ_{13}.

we get that

$$
\begin{aligned}
Y_e T_e T_e &= (\bar{3},3,1)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B3)}\\
Y_e T_e T_\nu &= (\bar{3},3,1)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B4)}\\
Y_e T_\nu T_e &= (\bar{3},3,1)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B5)}\\
Y_e T_\nu T_\nu &= (\bar{3},3,1)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B6)}\\
Y_e \mathrm{Tr}[Y_e^\dagger Y_e] T_e &= (\bar{3},3,1)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B7)}\\
Y_e \mathrm{Tr}[Y_\nu^\dagger Y_\nu] T_e &= (\bar{3},3,1)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B8)}\\
Y_e \mathrm{Tr}[Y_e^\dagger Y_e] T_\nu &= (\bar{3},3,1)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B9)}\\
Y_e \mathrm{Tr}[Y_\nu^\dagger Y_\nu] T_\nu &= (\bar{3},3,1)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},3,1)\ , && \text{(B10)}\\
Y_e \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] &= (\bar{3},3,1)\otimes(1,1,1)=(\bar{3},3,1)\ , && \text{(B11)}\\
Y_e \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] &= (\bar{3},3,1)\otimes(1,1,1)=(\bar{3},3,1)\ , && \text{(B12)}\\
\text{and}\quad Y_e \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu] &= (\bar{3},3,1)\otimes(1,1,1)=(\bar{3},3,1) && \text{(B13)}
\end{aligned}
$$

are the only allowed combinations of five spurion fields that can appear on the RHS of $\dot{Y}_e$ at second order. Hence we can write the most general form of the second order contributions to $\dot{Y}_e$ as

$$
\begin{aligned}
(4\pi)^2\,\dot{Y}_e\Big|_{\text{2-loop}} \sim\; & Y_e\left(\widetilde{d}_1 T_e T_e + \widetilde{d}_2 T_e T_\nu + \widetilde{d}_3 T_\nu T_e + \widetilde{d}_4 T_\nu T_\nu\right)\\
+\; & Y_e\left(\widetilde{d}_5 \mathrm{Tr}[Y_e^\dagger Y_e] T_e + \widetilde{d}_6 \mathrm{Tr}[Y_\nu^\dagger Y_\nu] T_e + \widetilde{d}_7 \mathrm{Tr}[Y_e^\dagger Y_e] T_\nu + \widetilde{d}_8 \mathrm{Tr}[Y_\nu^\dagger Y_\nu] T_\nu\right)\\
+\; & Y_e\left(\widetilde{d}_9 \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + \widetilde{d}_{10} \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + \widetilde{d}_{11} \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu]\right)\\
+\; & Y_e\left(\widetilde{d}_{12} T_e + \widetilde{d}_{13} T_\nu + \widetilde{d}_{14} \mathrm{Tr}[Y_e^\dagger Y_e] + \widetilde{d}_{15} \mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right) + \widetilde{d}_{16} Y_e\ . \qquad \text{(B14)}
\end{aligned}
$$

The extra factor of $(4\pi)^2$ is there since we are considering 2-loop contributions. We rewrite Eq. (B14), using the definitions of T_e, T_ν from Table I, as

$$
\begin{aligned}
(4\pi)^2\,\dot{Y}_e\Big|_{\text{2-loop}} =\; & Y_e\left(d_1 Y_e^\dagger Y_e Y_e^\dagger Y_e + d_2 Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu + d_3 Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e + d_4 Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu\right)\\
+\; & Y_e\left(d_5 Y_e^\dagger Y_e \mathrm{Tr}[Y_e^\dagger Y_e] + d_6 Y_e^\dagger Y_e \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + d_7 Y_\nu^\dagger Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e] + d_8 Y_\nu^\dagger Y_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right)\\
+\; & Y_e\left(d_9 \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + d_{10} \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + d_{11} \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu]\right)\\
+\; & Y_e\left(d_{12} Y_e^\dagger Y_e + d_{13} Y_\nu^\dagger Y_\nu + d_{14} \mathrm{Tr}[Y_e^\dagger Y_e] + d_{15} \mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right) + d_{16} Y_e\ , \qquad \text{(B15)}
\end{aligned}
$$

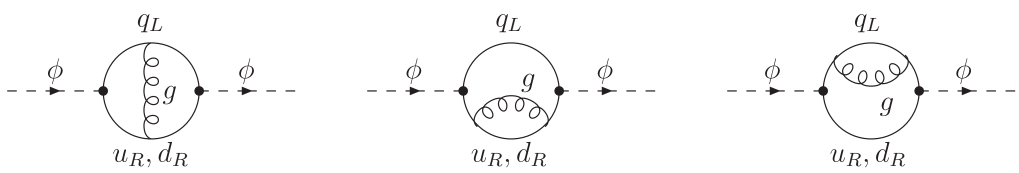

FIG. 6: Example of diagrams at 2-loop with gluon contributions, leading to terms proportional to $g_3^2\mathrm{Tr}[Y_U^\dagger Y_U]$ and $g_3^2\mathrm{Tr}[Y_D^\dagger Y_D]$ in d_{16}.

where d_1, $\cdots$, d_{11} are expected to be $\mathcal{O}(1)$ numbers. We have not written the terms of the form $\mathrm{Tr}[Y_i^\dagger Y_i]\cdot\mathrm{Tr}[Y_j^\dagger Y_j]$, $i,j\in\{e,\nu\}$, since such terms cannot be generated at 2-loop order. Each of d_{12}, d_{13} is expected to be a linear function of g_i^2, λ, $\mathrm{Tr}[Y_U^\dagger Y_U]$, $\mathrm{Tr}[Y_D^\dagger Y_D]$ and can be written, in general, as

$$d_i = d_i^{g_1} g_1^2 + d_i^{g_2} g_2^2 + d_i^\lambda \lambda + d_i^U \mathrm{Tr}[Y_U^\dagger Y_U] + d_i^D \mathrm{Tr}[Y_D^\dagger Y_D] \qquad (i\in\{12,13\})\ . \tag{B16}$$

Unlike the 1-loop case, Higgs self-coupling can appear at 2-loop order via diagrams like the one shown in Fig. 5. Since the leptons are singlets under $\mathrm{SU}(3)_C$, g_3^2 cannot be present in d_{12} and d_{13}. As before, d_{12}^x, d_{13}^x are expected to be of $\mathcal{O}(1)$. d_{14}, d_{15} must originate from a diagram containing complete lepton loop in Higgs self-energy correction and hence cannot contain λ or g_3^2. Hence we write

$$d_i = d_i^{g_1} g_1^2 + d_i^{g_2} g_2^2 \qquad (i\in\{14,15\})\ , \tag{B17}$$

where d_{14}^x, d_{15}^x are to be of $\mathcal{O}(1)$. $\mathrm{Tr}[Y_U^\dagger Y_U]$ or $\mathrm{Tr}[Y_D^\dagger Y_D]$ cannot be present in d_{14} and d_{15}.

Let us now consider the quantity d_{16}, which is independent of the spurion fields and must be a function linear in $\mathrm{Tr}[Y_U^\dagger Y_U Y_U^\dagger Y_U]$, $\mathrm{Tr}[Y_D^\dagger Y_D Y_D^\dagger Y_D]$ and quadratic in g_i^2, $\mathrm{Tr}[Y_U^\dagger Y_U]$, $\mathrm{Tr}[Y_D^\dagger Y_D]$ and λ. In its most general form, it can be expressed as

$$\begin{aligned} d_{16} \;=\;& d_{16}^{UU}\mathrm{Tr}[Y_U^\dagger Y_U Y_U^\dagger Y_U] + d_{16}^{DD}\mathrm{Tr}[Y_D^\dagger Y_D Y_D^\dagger Y_D] + d_{16}^{UD}\mathrm{Tr}[Y_U^\dagger Y_U Y_D^\dagger Y_D] \\ &+ \left(d_{16}^{g_1U} g_1^2 + d_{16}^{g_2U} g_2^2 + d_{16}^{g_3U} g_3^2\right)\mathrm{Tr}[Y_U^\dagger Y_U] + \left(d_{16}^{g_1D} g_1^2 + d_{16}^{g_2D} g_2^2 + d_{16}^{g_3D} g_3^2\right)\mathrm{Tr}[Y_D^\dagger Y_D] \\ &+ \left(d_{16}^{g_1\lambda} g_1^2 + d_{16}^{g_2\lambda} g_2^2\right)\lambda + d_{16}^{g_1} g_1^4 + d_{16}^{g_2} g_2^4 + d_{16}^{g_1g_2} g_1^2 g_2^2\ , \end{aligned} \tag{B18}$$

where all the coefficients d_{16}^x are expected to be $\mathcal{O}(1)$ numbers. Unlike the case of first order evolution equation, here g_3^2 can appear at 2-loop since quarks have color charges. For example, diagrams shown in Fig. 6 will contribute terms proportional to $g_3^2\mathrm{Tr}[Y_U^\dagger Y_U]$ and $g_3^2\mathrm{Tr}[Y_D^\dagger Y_D]$. However, terms proportional to g_3^4 cannot be present. As can be checked, here we cannot have terms proportional to $\lambda\mathrm{Tr}[Y_U^\dagger Y_U]$ or $\lambda\mathrm{Tr}[Y_D^\dagger Y_D]$, while terms containing λg_1^2,λg_2^2 can contribute. Examples of diagrams giving rise to such terms are shown in Fig. 7. There cannot exist any term proportional to λg_3^2 or λ^2 in this case.

Having written the most general form of second order contributions to $\dot{Y}_e$, we consider the fact that $\mathrm{Tr}[Y_i^\dagger Y_i]$ ($i\in\{e,\nu,U,D\}$) can only come from a complete fermionic loop in the

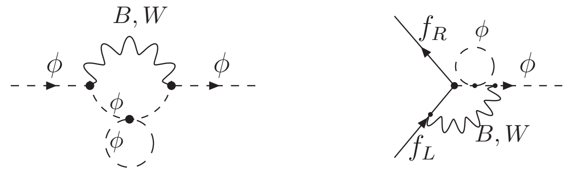

FIG. 7: Example of diagrams at 2-loop giving rise to λg_1^2,λg_2^2 terms in d_{16}.

Higgs self-energy correction, as already stated in Section III A and shown in Fig. 1. Hence, we can write the ratios as

$$\begin{aligned} d_5 : d_6 : d_{12}^U : d_{12}^D &= 1 : r : 3 : 3 \ , \\ d_7 : d_8 : d_{13}^U : d_{13}^D &= 1 : r : 3 : 3 \ , \end{aligned} \tag{B19}$$

where r for Type-I and Type-III seesaw is defined in Eq. (4.3) for SM and in Eq. (4.14) for MSSM. Hence, we can write

$$\begin{aligned} d_5 \mathrm{Tr}[Y_e^\dagger Y_e] + d_6 \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + d_{12}^U \mathrm{Tr}[Y_U^\dagger Y_U] + d_{12}^D \mathrm{Tr}[Y_D^\dagger Y_D] &\rightarrow d_{12}^T T \ , \\ d_7 \mathrm{Tr}[Y_e^\dagger Y_e] + d_8 \mathrm{Tr}[Y_\nu^\dagger Y_\nu] + d_{13}^U \mathrm{Tr}[Y_U^\dagger Y_U] + d_{13}^D \mathrm{Tr}[Y_D^\dagger Y_D] &\rightarrow d_{13}^T T \ , \end{aligned}$$

where T is defined in Eq. (3.13) and d_{13}^T, d_{14}^T are expected to be of $\mathcal{O}(1)$. Thus the most general form of $\dot{Y}_e$ becomes

$$\begin{aligned} (4\pi)^2 \, \dot{Y}_e \Big|_{\text{2-loop}} \ = \ & Y_e \left(d_1 Y_e^\dagger Y_e Y_e^\dagger Y_e + d_2 Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu + d_3 Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e + d_4 Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu \right) \\ + \ & Y_e \left(d_9 \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + d_{10} \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + d_{11} \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu] \right) \\ + \ & Y_e \left(d_{12}^{g_1} g_1^2 + d_{12}^{g_2} g_2^2 + d_{12}^\lambda \lambda + d_{12}^T T \right) Y_e^\dagger Y_e + Y_e \left(d_{13}^{g_1} g_1^2 + d_{14}^{g_2} g_2^2 + d_{13}^\lambda \lambda + d_{13}^T T \right) Y_\nu^\dagger Y_\nu \\ + \ & Y_e \left(d_{14} \mathrm{Tr}[Y_e^\dagger Y_e] + d_{15} \mathrm{Tr}[Y_\nu^\dagger Y_\nu] \right) + d_{16} Y_e \ . \end{aligned} \tag{B20}$$

2. 2-loop running of Y_ν

Let us now consider the second order terms arising in the RGE of Y_ν. Considering Table I, the transformation rules in Eqs. (3.2, B1), and the transformation properties in Eq. (B2),

we get that

$$
\begin{aligned}
Y_\nu T_e T_e &= (\bar{3},1,3)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B21)}\\
Y_\nu T_e T_\nu &= (\bar{3},1,3)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B22)}\\
Y_\nu T_\nu T_e &= (\bar{3},1,3)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B23)}\\
Y_\nu T_\nu T_\nu &= (\bar{3},1,3)\otimes(8,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B24)}\\
Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e]T_e &= (\bar{3},1,3)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B25)}\\
Y_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu]T_e &= (\bar{3},1,3)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B26)}\\
Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e]T_\nu &= (\bar{3},1,3)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B27)}\\
Y_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu]T_\nu &= (\bar{3},1,3)\otimes(1,1,1)\otimes(8,1,1)\ni(\bar{3},1,3)\,, && \text{(B28)}\\
Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] &= (\bar{3},1,3)\otimes(1,1,1)=(\bar{3},1,3)\,, && \text{(B29)}\\
Y_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] &= (\bar{3},1,3)\otimes(1,1,1)=(\bar{3},1,3)\,, && \text{(B30)}\\
\text{and}\quad Y_\nu \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu] &= (\bar{3},1,3)\otimes(1,1,1)=(\bar{3},1,3) && \text{(B31)}
\end{aligned}
$$

are the only allowed combinations of five spurion fields that can appear on the RHS of $\dot{Y}_\nu$ at second order. Hence, similar to $\dot{Y}_e$, we can write the most general form of the second order contributions to $\dot{Y}_\nu$ as

$$
\begin{aligned}
(4\pi)^2\,\dot{Y}_\nu\Big|_{\text{2-loop}} \sim\; & Y_\nu\left(\widetilde{f}_1 T_e T_e + \widetilde{f}_2 T_e T_\nu + \widetilde{f}_3 T_\nu T_e + \widetilde{f}_4 T_\nu T_\nu\right)\\
+\; & Y_\nu\left(\widetilde{f}_5 \mathrm{Tr}[Y_e^\dagger Y_e]T_e + \widetilde{f}_6 \mathrm{Tr}[Y_\nu^\dagger Y_\nu]T_e + \widetilde{f}_7 \mathrm{Tr}[Y_e^\dagger Y_e]T_\nu + \widetilde{f}_8 \mathrm{Tr}[Y_\nu^\dagger Y_\nu]T_\nu\right)\\
+\; & Y_\nu\left(\widetilde{f}_9 \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + \widetilde{f}_{10} \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + \widetilde{f}_{11} \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu]\right)\\
+\; & Y_\nu\left(\widetilde{f}_{12} T_e + \widetilde{f}_{13} T_\nu + \widetilde{f}_{14} \mathrm{Tr}[Y_e^\dagger Y_e] + \widetilde{f}_{15} \mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right) + \widetilde{f}_{16} Y_e\,. \qquad \text{(B32)}
\end{aligned}
$$

The above equation can be written in a simple form using the definitions of T_e, T_ν from Table I and the ratio of the coefficient of the traces, as done in case of $\dot{Y}_e$, to give

$$
\begin{aligned}
(4\pi)^2\,\dot{Y}_\nu\Big|_{\text{2-loop}} =\; & Y_\nu\left(f_1 Y_e^\dagger Y_e Y_e^\dagger Y_e + f_2 Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu + f_3 Y_\nu^\dagger Y_\nu Y_e^\dagger Y_e + f_4 Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu\right)\\
+\; & Y_\nu\left(f_9 \mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + f_{10} \mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + f_{11} \mathrm{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu]\right)\\
+\; & Y_\nu\left(f_{12}^{g_1} g_1^2 + f_{13}^{g_2} g_2^2 + f_{12}^{\lambda}\lambda + f_{12}^{T} T\right) Y_e^\dagger Y_e + Y_\nu\left(f_{13}^{g_1} g_1^2 + f_{13}^{g_2} g_2^2 + f_{13}^{\lambda}\lambda + f_{13}^{T} T\right) Y_\nu^\dagger Y_\nu\\
+\; & Y_\nu\left(f_{14} \mathrm{Tr}[Y_e^\dagger Y_e] + f_{15} \mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right) + f_{16} Y_\nu\,, \qquad \text{(B33)}
\end{aligned}
$$

with T defined in Eq. (3.13). Here, $f_{1,\cdots,4}$, $f_{9,\cdots,11}$, f_{12}^x and f_{13}^x are expected to be $\mathcal{O}(1)$ numbers. f_i (i=14,15,16) will have similar forms as d_i (i=14,15,16), as given in Eqs. (B17) and (B18) respectively, with all f_i^x (i=14,15,16) being $\mathcal{O}(1)$ quantities. As before, we have not written the terms of the form $\mathrm{Tr}[Y_i^\dagger Y_i]\cdot\mathrm{Tr}[Y_j^\dagger Y_j]$, $i,j\in\{e,\nu,U,D\}$, since such terms cannot be generated at 2-loop.

3. 2-loop running of M_ν

Next, we discuss the second order contribution to $\dot{M}_\nu$. Using Table I and the SU(3) algebra given in Eqs. (3.22) and (B1), we obtain that

$$
\begin{aligned}
M_\nu T'_\nu T'_\nu &= (1,1,6)\otimes(1,1,8)\otimes(1,1,8) \ni (1,1,6)\,, && \text{(B34)}\\
M_\nu \left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right) &= (1,1,6)\otimes(\bar{3},1,3)\otimes(8\oplus 1,1,1)\otimes(3,1,\bar{3}) \ni (1,1,6)\,, && \text{(B35)}\\
T'^T_\nu M_\nu T'_\nu &= (1,1,8)\otimes(1,1,6)\otimes(1,1,8) \ni (1,1,6)\,, && \text{(B36)}\\
M_\nu \mathrm{Tr}[Y_e^\dagger Y_e] T'_\nu &= (1,1,6)\otimes(1,1,1)\otimes(1,1,8) \ni (1,1,6)\,, && \text{(B37)}\\
\text{and}\quad M_\nu \mathrm{Tr}[Y_\nu^\dagger Y_\nu] T'_\nu &= (1,1,6)\otimes(1,1,1)\otimes(1,1,8) \ni (1,1,6) && \text{(B38)}
\end{aligned}
$$

are the only combinations of five spurion fields that can contribute to $\dot{M}_\nu$. The term in Eq. (B35), not present in Table I, is an allowed combination at second order. Here we have considered the fact that M_ν couples only to the right-handed neutrinos and hence $\dot{M}_\nu$ cannot contain trace of four spurions at second order. Apart from the above terms, there will also be terms with three spurions and two other couplings in the theory, transforming trivially under $\mathrm{G_{LF}}$. Terms containing one spurion and four other couplings are also allowed at this order. However, M_ν being coupled to right-handed neutrinos alone, $\dot{M}_\nu$ will not contain terms proportional to trace of four $Y_{U,D}$ and also no g_1^4 or λ^2. If the right-handed neutrinos are singlets under the gauge group, as is the case for Type-I seesaw, they will not have any $\mathrm{SU}(2)_L$ or $\mathrm{SU}(3)_C$ charges and hence terms proportional to g_2^4, g_3^4 be absent. However, for Type-III seesaw scenario these are triplet under $\mathrm{SU}(2)_L$ and hence g_2^4 contribution is expected to be there.

Finally, symmetrizing over the $\mathrm{O}(3)_{\nu_R}$ indices, the most general form of the 2-loop contribution to $\dot{M}_\nu$ can be written as

$$
\begin{aligned}
(4\pi)^2\, \dot{M}_\nu\Big|_{\text{2-loop}} &= \frac{\widetilde{h}_1}{2}\left(M_\nu\,(T'_\nu T'_\nu) + (T'_\nu T'_\nu)^T M_\nu\right) + \frac{h_2}{2}\left(M_\nu\left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right) + \left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right)^T M_\nu\right)\\
&+ \widetilde{h}_3 T'^T_\nu M_\nu T'_\nu + \frac{1}{2}\left(\widetilde{h}'_4 \mathrm{Tr}[Y_e^\dagger Y_e] + \widetilde{h}''_4 \mathrm{Tr}[Y_\nu^\dagger Y_\nu]\right)\left(M_\nu T'_\nu + T'^T_\nu M_\nu\right)\\
&+ \frac{\widetilde{h}_4}{2}\left(M_\nu T'_\nu + T'^T_\nu M_\nu\right) + \widetilde{h}_5 g_2^4 M_\nu\,. \qquad \text{(B39)}
\end{aligned}
$$

Eq. (B39) can be simplified using the definition of T'_ν and the fact that terms proportional to $\mathrm{Tr}[Y_i^\dagger Y_i]$ ($i \in \{e,\nu,U,D\}$) appear only in the combination T, defined in Eq. (3.13), to get

$$
\begin{aligned}
(4\pi)^2\, \dot{M}_\nu\Big|_{\text{2-loop}} &= h_1\left(M_\nu\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) + \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)^T M_\nu\right)\\
&+ h_2\left(M_\nu\left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right) + \left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right)^T M_\nu\right) + h_3\left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\left(Y_\nu Y_\nu^\dagger\right)\\
&+ h_4\left(M_\nu\left(Y_\nu Y_\nu^\dagger\right) + \left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\right) + h_5 g_2^4 M_\nu\,, \qquad \text{(B40)}
\end{aligned}
$$

where h_1, h_2, h_3 and h_5 are expected to be $\mathcal{O}(1)$ numbers in general. For Type-I seesaw, $h_5 = 0$. In writing Eq. (B40), we have considered the fact that terms with $\mathrm{Tr}[Y_i^\dagger Y_i]\,\cdot$

$\text{Tr}[Y_j^\dagger Y_j](i,j\in\{e,\nu\})$ cannot be present at 2-loop. h_4 can in general be a linear function of g_1^2, g_2^2, λ and T and be given by

$$h_4 = h_4^{g_1} g_1^2 + h_4^{g_2} g_2^2 + h_4^{\lambda}\lambda + h_4^T T \ , \tag{B41}$$

where all h_4^x must be of $\mathcal{O}(1)$. In writing Eq. (B40) we have used the symmetry property of M_ν: $M_\nu^T = M_\nu$. Leptons and Higgs, being singlets under $\text{SU}(3)_C$, h_4 will not involve g_3^2.

As before, we expect Eq. (B40) to give the right-handed projection of $\dot{M}_\nu$ only. The most general form of $\dot{M}_\nu$ will be given by

$$\begin{aligned}
(4\pi)^2\,\dot{M}_\nu\Big|_{\text{2-loop}} &= h_1\Big[\Big(M_\nu\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) + \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)^T M_\nu\Big)P_R + \Big(M_\nu\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)^T + \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)M_\nu\Big)P_L\Big]\\
&+ h_2\Big[\Big(M_\nu\left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right) + \left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right)^T M_\nu\Big)P_R + \Big(M_\nu\left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right)^T + \left(Y_\nu Y_e^\dagger Y_e Y_\nu^\dagger\right)M_\nu\Big)P_L\Big]\\
&+ h_3\Big[\left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\left(Y_\nu Y_\nu^\dagger\right)P_R + \left(Y_\nu Y_\nu^\dagger\right)M_\nu\left(Y_\nu Y_\nu^\dagger\right)^T P_L\Big]\\
&+ h_4\Big[\Big(M_\nu\left(Y_\nu Y_\nu^\dagger\right) + \left(Y_\nu Y_\nu^\dagger\right)^T M_\nu\Big)P_R + \Big(M_\nu\left(Y_\nu Y_\nu^\dagger\right)^T + \left(Y_\nu Y_\nu^\dagger\right)M_\nu\Big)P_L\Big] + h_5 g_2^4 M_\nu\ .
\end{aligned} \tag{B42}$$

4. 2-loop running of the left-handed mass m_ν at $\mu < M_R$

At the energy scale $\mu < M_R$, the flavor symmetry group is G'_{LF} and $Y_e(\bar{3},3)$, $m_\nu(6,1)$ are the only spurions in the theory. Let us first consider the running of Y_e at this scale which can be obtained from Eq. (B20) simply by setting the coefficients of terms containing Y_ν to zero and we have

$$\begin{aligned}
(4\pi)^2\,\dot{Y}_e\Big|_{\text{2-loop}} &= Y_e\left(d_1 Y_e^\dagger Y_e Y_e^\dagger Y_e + d_9\text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + \left(d_{12}^{g_1} g_1^2 + d_{12}^{g_2} g_2^2 + d_{12}^{\lambda}\lambda + d_{12}^T T\right)Y_e^\dagger Y_e\right)\\
&+ Y_e\left(d_{14}^{g_1} g_1^2 + d_{14}^{g_2} g_2^2\right)\text{Tr}[Y_e^\dagger Y_e] + d_{16} Y_e\ ,
\end{aligned} \tag{B43}$$

where d_{16} is given by Eq. (B18). d_1, d_9, d_{12}^x and d_{14}^x are expected to be $\mathcal{O}(1)$ numbers.

Now we consider the running of the left-handed mass m_ν. Using Table I, the transformation rules in Eq. (B2) and the SU(3) algebra given in Eqs. (3.22, B1) we get the second order contributions to $\dot{m}_\nu$ to contain the following combinations of five spurions:

$$m_\nu T_e T_e = (6,1)\otimes(8,1)\otimes(8,1)\ni(6,1)\ , \tag{B44}$$

$$T_e^T m_\nu T_e = (8,1)\otimes(6,1)\otimes(8,1)\ni(6,1)\ , \tag{B45}$$

$$m_\nu\text{Tr}[Y_e^\dagger Y_e]T_e = (6,1)\otimes(1,1)\otimes(8,1)\ni(6,1)\ , \tag{B46}$$

$$\text{and}\quad m_\nu\text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] = (6,1)\otimes(1,1) = (6,1)\ , \tag{B47}$$

where the terms proportional to $\text{Tr}[Y_e^\dagger Y_e]\cdot\text{Tr}[Y_e^\dagger Y_e]$ are to be removed since such terms cannot arise at 2-loop. Finally, symmetrizing over the $\text{SU}(3)_{l_L}$ indices, we write down the

most general form of $\dot{m}_\nu$ at second order as

$$\begin{aligned}(4\pi)^2\,\dot{m}_\nu\big|_{\text{2-loop}} &= r_1\left(m_\nu\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)+\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)^T m_\nu\right)+r_2\left(Y_e^\dagger Y_e\right)^T m_\nu\left(Y_e^\dagger Y_e\right)\\ &+ r_3\mathrm{Tr}[Y_e^\dagger Y_e]\left(m_\nu\left(Y_e^\dagger Y_e\right)+\left(Y_e^\dagger Y_e\right)^T m_\nu\right)+r_4\mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e]m_\nu\\ &+ r_5\left(m_\nu\left(Y_e^\dagger Y_e\right)+\left(Y_e^\dagger Y_e\right)^T m_\nu\right)+r_6 m_\nu\ , \end{aligned}\tag{B48}$$

where r_1, r_2, r_3, r_4 are expected to be of $\mathcal{O}(1)$, while the general forms of r_5, r_6 are

$$r_5 = r_5^U\mathrm{Tr}[Y_U^\dagger Y_U]+r_5^D\mathrm{Tr}[Y_D^\dagger Y_D]+r_5^{g_1}g_1^2+r_5^{g_2}g_2^2+r_5^\lambda\lambda\ ,\tag{B49}$$

$$\begin{aligned}r_6 &= r_6^{UU}\mathrm{Tr}[Y_U^\dagger Y_U Y_U^\dagger Y_U]+r_6^{DD}\mathrm{Tr}[Y_D^\dagger Y_D Y_D^\dagger Y_D]+r_6^{UD}\mathrm{Tr}[Y_U^\dagger Y_U Y_D^\dagger Y_D]\\ &+ \left(r_6^{g_1U}g_1^2+r_6^{g_2U}g_2^2+r_6^{g_3U}g_3^2\right)\mathrm{Tr}[Y_U^\dagger Y_U]+\left(r_6^{g_1D}g_1^2+r_6^{g_2D}g_2^2+r_6^{g_3D}g_3^2\right)\mathrm{Tr}[Y_D^\dagger Y_D]\\ &+ \left(r_6^{g_1\lambda}g_1^2+r_6^{g_2\lambda}g_2^2\right)\lambda+r_6^\lambda\lambda^2+r_6^{g_1}g_1^4+r_6^{g_2}g_2^4+r_6^{g_1g_2}g_1^2g_2^2\ ,\end{aligned}\tag{B50}$$

with all r_5^x, r_6^x being expected to be $\mathcal{O}(1)$ numbers. We can further simplify by considering the fact that terms proportional to $\mathrm{Tr}[Y_i^\dagger Y_i](i\in\{e,U,D\})$ come through a complete fermion loop in Higgs self-energy corrections and hence we must have

$$r_3\mathrm{Tr}[Y_e^\dagger Y_e]+r_5^U\mathrm{Tr}[Y_U^\dagger Y_U]+r_5^D\mathrm{Tr}[Y_D^\dagger Y_D]\rightarrow r_5^T\,T'\ ,$$

where T' is defined in Eq. (3.37) and r_5^T is of $\mathcal{O}(1)$. So the 2-loop contribution to $\dot{m}_\nu$ becomes

$$\begin{aligned}(4\pi)^2\,\dot{m}_\nu\big|_{\text{2-loop}} &= r_1\left(m_\nu\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)+\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)^T m_\nu\right)+r_2\left(Y_e^\dagger Y_e\right)^T m_\nu\left(Y_e^\dagger Y_e\right)\\ &+ r_4\mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e]m_\nu+r_5'\left(m_\nu\left(Y_e^\dagger Y_e\right)+\left(Y_e^\dagger Y_e\right)^T m_\nu\right)+r_6 m_\nu\ ,\end{aligned}\tag{B51}$$

with

$$r_5' = r_5^T\,T'+r_5^{g_1}g_1^2+r_5^{g_2}g_2^2+r_5^\lambda\lambda\ .\tag{B52}$$

5. Results

First, let us consider the case of the SM. Second order contributions to the RG evolution equations of Y_e, Y_ν, M_ν or m_ν are not available in the literature for right-handed neutrino extended SM (Type-I or Type-III) in general. In Ref. [24], the contribution to $\dot{m}_\nu$ proportional to r_2 in Eq. (B51), for Type-I seesaw, is presented that gives

$$r_2 = 2\ .\tag{B53}$$

Thus r_2 is of $\mathcal{O}(1)$, as expected. In the future, once a full calculation is done, it can be checked against our results.

Next, we move to the case of the MSSM. Unlike the case of SM, there are existing results for second order contributions in extended MSSM for Type-I seesaw [12], obtained from exact computations. In order to compare the results with the equations obtained above, we keep the following facts in mind:

- Higgs self-coupling λ is absent in MSSM, hence all terms proportional to λ will vanish.

- Terms with $Y_e^\dagger Y_e$ can only contain $\mathrm{Tr}[Y_e^\dagger Y_e]$ and $\mathrm{Tr}[Y_D^\dagger Y_D]$, while terms with $Y_\nu^\dagger Y_\nu$ can only contain $\mathrm{Tr}[Y_\nu^\dagger Y_\nu]$ and $\mathrm{Tr}[Y_U^\dagger Y_U]$. Moreover they will appear only in the combinations T_U and T_D, defined in Eqs.(4.12) and (4.13) respectively. Thus, the terms present will be $T_D Y_e^\dagger Y_e$ and $T_U Y_\nu^\dagger Y_\nu$.

- $\dot{Y}_e$ cannot have terms proportional to $\mathrm{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu]$ or $\mathrm{Tr}[Y_U^\dagger Y_U Y_U^\dagger Y_U]$. Similarly, $\dot{Y}_\nu$ cannot have terms proportional to $\mathrm{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e]$ and $\mathrm{Tr}[Y_D^\dagger Y_D Y_D^\dagger Y_D]$. Hence

$$d_{10} = 0 , \qquad d_{16}^{UU} = 0 , \qquad f_9 = 0 , \qquad f_{16}^{DD} = 0 . \tag{B54}$$

- Since Y_e couples to H_D only, $\dot{Y}_e$ cannot contain terms $g_i^2 \mathrm{Tr}[Y_\nu^\dagger Y_\nu]$ and $g_i^2 \mathrm{Tr}[Y_U^\dagger Y_U]$. Similarly, $\dot{Y}_\nu$ cannot have terms $g_i^2 \mathrm{Tr}[Y_e^\dagger Y_e]$ or $g_i^2 \mathrm{Tr}[Y_D^\dagger Y_D]$. Thus,

$$\begin{aligned} d_{15}^{g_1} &= d_{15}^{g_2} = 0 , \qquad d_{16}^{g_1 U} = d_{16}^{g_2 U} = d_{16}^{g_3 U} = 0 \\ f_{14}^{g_1} &= f_{14}^{g_2} = 0 , \qquad f_{16}^{g_1 D} = f_{16}^{g_2 D} = f_{16}^{g_3 D} = 0 . \end{aligned} \tag{B55}$$

- The right-handed Majorana mass M_ν couples only to the right-handed neutrinos which interacts with H_U, and not with H_D, and so h_4 in Eq. (B41) becomes

$$h_4 = h_4^{g_1} g_1^2 + h_4^{g_2} g_2^2 + h_4^T T_U . \tag{B56}$$

- Only H_U is involved in the definition of the effective left-handed Majorana mass m_ν, and hence we must have

$$T' = T'_U = \mathrm{Tr}[Y_e^\dagger Y_e] + 3\mathrm{Tr}[Y_U^\dagger Y_U] , \tag{B57}$$

  and r_6 in Eq. (B50) will have

$$r_6^{DD} = 0 , \qquad r_6^{g_1 D} = 0 , \qquad r_6^{g_2 D} = 0 , \qquad r_6^{g_3 D} = 0 . \tag{B58}$$

Let us now compare the coefficients with the values obtained with exact computation [12]. For Y_e evolution we get

$$\begin{aligned} d_1 &= -4 , \quad d_2 = 0 , \quad d_3 = -2 , \quad d_4 = -2 , \quad d_9 = -3 , \quad d_{11} = -1 , \\ d_{12}^{g_1} &= 0 , \quad d_{12}^{g_2} = 6 , \quad d_{12}^T = -3 , \quad d_{13}^{g_1} = 0 , \quad d_{13}^{g_2} = 0 , \quad d_{13}^T = -1 , \\ d_{14}^{g_1} &= \frac{6}{5} , \quad d_{14}^{g_2} = 0 , \quad d_{16}^{DD} = -9 , \quad d_{16}^{UD} = -3 , \quad d_{16}^{g_1 D} = -\frac{2}{5} , \\ d_{16}^{g_2 D} &= 0 , \quad d_{16}^{g_3 D} = 16 , \quad d_{16}^{g_1} = \frac{27}{2} , \quad d_{16}^{g_2} = \frac{15}{2} , \quad d_{16}^{g_1 g_2} = \frac{9}{5} . \end{aligned} \tag{B59}$$

Comparing the coefficients of $\dot{Y}_\nu$, we get

$$\begin{aligned}
f_1 &= -2\,, \quad f_2 = -2\,, \quad f_3 = 0\,, \quad f_4 = -4\,, \quad f_{10} = -3\,, \quad f_{11} = -1\,,\\
f_{12}^{g_1} &= \frac{6}{5}\,, \quad f_{12}^{g_2} = 0\,, \quad f_{12}^{T} = -1\,, \quad f_{13}^{g_1} = \frac{6}{5}\,, \quad f_{13}^{g_2} = 6\,, \quad f_{13}^{T} = -3\,,\\
f_{15}^{g_1} &= 0\,, \quad f_{15}^{g_2} = 0\,, \quad f_{16}^{UU} = -9\,, \quad f_{16}^{UD} = -3\,, \quad f_{16}^{g_1U} = \frac{4}{5}\,,\\
f_{16}^{g_2U} &= 0\,, \quad f_{16}^{g_3U} = 16\,, \quad f_{16}^{g_1} = \frac{207}{50}\,, \quad f_{16}^{g_2} = \frac{15}{2}\,, \quad f_{16}^{g_1g_2} = \frac{9}{5}\,.
\end{aligned} \tag{B60}$$

Finally, comparing the evolution of M_ν and m_ν at second order, we find the values of h_i and r_is to be

$$\begin{aligned}
h_1 &= -2\,, \quad h_2 = -2\,, \quad h_3 = 0\,, \quad h_4^{g_1} = \frac{6}{5}\,, \quad h_4^{g_2} = 6\,, \quad h_4^{T} = -2\,,\\
r_1 &= -2\,, \quad r_2 = 0\,, \quad r_4 = 0\,, \quad r_5^{g_1} = \frac{6}{5}\,, \quad r_5^{g_2} = 0\,, \quad r_5^{T} = -1\,,\\
r_6^{UU} &= -18\,, \quad r_6^{UD} = -6\,, \quad r_6^{g_1U} = \frac{8}{5}\,, \quad r_6^{g_2U} = 0\,, \quad r_6^{g_3U} = 32\,,\\
r_6^{g_1} &= \frac{207}{25}\,, \quad r_6^{g_2} = 15\,, \quad r_6^{g_1g_2} = \frac{18}{5}\,.
\end{aligned} \tag{B61}$$

As we can see from Eqs. (B59), (B60), and (B61), there are a few zeros. If supersymmetry is not broken, one has $r_2 = 0$ in MSSM Type-I seesaw [24]. However, the remaining zeros cannot be explained using spurion techniques. There are also some quantities which are not of $\mathcal{O}(1)$, namely $d_{12}^{g_2}, d_{16}^{DD}, d_{16}^{g_3D}, d_{16}^{g_1}, d_{16}^{g_2}$, $f_{13}^{g_2}, f_{16}^{UU}, f_{16}^{g_3U}, f_{16}^{g_2}$, $r_6^{UU}, r_6^{UD}, r_6^{g_3U}, r_6^{g_1}$ and $r_6^{g_2}$. Of them, $x_i^{UU}, x_i^{UD}, x_i^{DD}, x_i^{g_3U}, x_i^{g_3D}$ can be large due to color factors, while the remaining become large because of the effect of gauge interactions.

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